HOJA 1 DE EJERCICIOS PROPUESTOS
UNIDAD 5: **INTEGRALES INDEFINIDAS**

Ejercicio 1: Calcular las siguientes integrales indefinidas:

a) $I=\int x^4 dx$	b) $I=\int \frac{4}{3}x^2 dx$	c) $I=\int\left(4x^6+2x^3-13x\right)dx$
d) $I=\int \frac{1}{x^2}dx$	e) $I=\int \frac{-2}{7x^4}dx$	f) $I=\int \frac{2}{5}x^{\frac{2}{5}}dx$
g) $I=\int \sqrt{x}dx$	h) $I=\int \sqrt[5]{x^3}dx$	j) $I=\int \frac{1}{x^{-3/2}}dx$
k) $I=\int \frac{1}{\sqrt{x}}dx$	l) $I=\int \frac{1}{\sqrt[4]{x^5}}dx$	m) $I=\int x\left(x^2+3\right)^4 dx$
n) $I=\int\left(x^5-2x^3+1\right)\cdot\left(5x^4-6x^2\right)dx$	ñ) $I=\int\left(4x^6+2x^3\right)\cdot\left(4x^5+x^2\right)dx$	o) $I=\int \frac{3x^2-2}{\left(x^3-2x\right)^7}dx$
p) $I=\int\sqrt{x^8}\cdot\sqrt[3]{\left(x^5+1\right)^2}\cdot\sqrt[4]{x^5+1}\cdot\sqrt{x^5+1}dx$	q) $I=\int\left(x^2-2x\right)\cdot\left(x-1\right)dx$	r) $I=\int \frac{4x-2}{\left(x^2-x+1\right)^7}dx$
s) $I=\int \frac{\sqrt[4]{\left(x+1\right)^3}}{\left(x+1\right)^{1/4}}dx$	t) $I=\int \text{sen } x\cdot\cos^2 x\, dx$	u) $I=\int \frac{sen\ x}{\cos^3 x}dx$

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a) $\int x^4 dx = \frac{x^5}{5}+C$

b) $\int \frac{4}{3}x^2 dx = \frac{4}{3}\cdot\frac{x^3}{3}+C = \frac{4x^3}{9}+C$

c) $\int\left(4x^6+2x^3-13x\right)dx = 4\frac{x^7}{7}+\frac{2x^4}{4}-13\cdot\frac{x^2}{2}+C$

d) $\int \frac{1}{x^2}dx = -\frac{1}{x}+C$

e) $\int \frac{-2}{7x^4}dx = -\frac{2}{7}\int x^{-4}dx = -\frac{2}{7}\cdot\frac{x^{-4+1}}{-4+1}+C = -\frac{2}{7}\cdot\frac{x^{-3}}{-3}+C$

f) $\int \frac{2}{5}x^{2/5} = \frac{2}{21}\cdot\frac{1}{x^3}+C$

$= \frac{2}{5}\int x^{2/5}dx = \frac{2}{5}\cdot\frac{x^{\frac{2}{5}+1}}{\frac{2}{5}+1}+C = \frac{2}{\cancel{5}}\cdot\frac{x^{7/5}}{7/\cancel{5}}+C = \frac{2}{7}x^{7/5}+C$

g) $\int\sqrt{x}\,dx = \int x^{1/2}dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}$

$= \frac{x^{3/2}}{3/2}+C = \frac{2}{3}\sqrt{x^3}+C$

h) $\int\sqrt[5]{x^3}\,dx = \int x^{3/5}dx = \frac{x^{\frac{3}{5}+1}}{\frac{3}{5}+1}+C = \frac{x^{8/5}}{8/5}+C = \frac{5}{8}\sqrt[5]{x^8}+C$

j) $\displaystyle\int \frac{1}{x^{-3/2}}\,dx = \int x^{3/2}\,dx = \frac{x^{3/2+1}}{\frac{3}{2}+1} + C = \frac{x^{5/2}}{5/2} + C$

$\displaystyle = \frac{2}{5}x^{5/2} + C$

k) $\displaystyle\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{-1/2+1}}{-\frac{1}{2}+1} + C = \frac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$

l) $\displaystyle\int \frac{1}{\sqrt[4]{x^5}}\,dx = \int x^{-5/4}\,dx = \frac{x^{-5/4+1}}{-\frac{5}{4}+1} + C = \frac{x^{-1/4}}{-1/4} = \frac{-4}{\sqrt[4]{x}} + C$

m) $\displaystyle\frac{1}{2}\int 2x\,(x^2+3)^4\,dx = \frac{1}{2}\frac{(x^2+3)^5}{5} + C = \frac{(x^2+3)^5}{10} + C$

n) $\displaystyle\int \underbrace{(x^5-2x^3+1)}_{f(x)} \cdot \underbrace{(5x^4-6x^2)\,dx}_{f'(x)\,dx} = \frac{(x^5-2x^3+1)^2}{2} + C$

ñ) $\displaystyle\frac{1}{6}\int 6\underbrace{(4x^6+2x^3)}_{f(x)} \cdot \underbrace{(4x^5+x^2)\,dx}_{f'(x)\,dx} = \frac{(4x^6+2x^3)^2}{2} \cdot \frac{1}{6} + C$

$24x^5+6x^2 = 6(4x^5+x^2)$

o) $\displaystyle\int \frac{3x^2-2}{x^3-2x}\,dx = \ln|x^3-2x| + C$

p) $\displaystyle\int \sqrt{x^8} \cdot \sqrt[3]{(x^5+1)^2} \cdot \sqrt[4]{x^5+1} \cdot \sqrt{x^5+1}\,dx = \int x^{8/2} \cdot (x^5+1)^{2/3} \cdot (x^5+1)^{1/4} (x^5+1)^{1/2}$

$\displaystyle = \int x^{8/2} \cdot (x^5+1)^{\frac{2}{3}+\frac{1}{4}+\frac{1}{2}}\,dx = \int x^4 \cdot (x^5+1)^{17/12}\,dx$

$\displaystyle = \frac{1}{5} \cdot \frac{(x^5+1)^{\frac{17}{12}+1}}{\frac{17}{12}+1} + C = \frac{1}{5} \cdot \frac{(x^5+1)^{\frac{29}{12}}}{\frac{29}{12}} + C$

q) $\displaystyle\int (x^2-2x)\cdot(x-1)\,dx = \frac{(x^2-2x)^2}{2} \cdot \frac{1}{2} + C = \frac{1}{4}(x^2-2x)^2 + C$

r) $\displaystyle\int \frac{4x-2}{(x^2-x+1)^7}\,dx = 2\int \frac{2x-1}{(x^2-x+1)^7}\,dx = 2\int (x^2-x+1)^{-7} \cdot (2x-1)\,dx$

$\displaystyle = 2 \cdot \frac{(x^2-x+1)^{-7+1}}{-7+1} + C = 2 \cdot \frac{(x^2-x+1)^{-6}}{-6} = \frac{-1}{3\cdot(x^2-x+1)^6} + C$

s) $\int \frac{\sqrt[4]{(x+1)^3}}{(x+1)^{1/4}}\,dx = \int (x+1)^{3/4-1/4}\,dx = \int (x+1)^{1/2}\,dx = \frac{(x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{2}{3}(x+1)^{3/2} + C = \frac{2}{3}\sqrt{(x+1)^3} + C$

t) $\int \text{sen}\,x \cdot \cos^2 x\,dx = -\frac{\cos^3 x}{3} + C$

u) $\int \frac{\text{sen}\,x}{\cos^3 x}\,dx = \int \cos^{-3}x \cdot \text{sen}\,x\,dx = -\frac{(\cos x)^{-3+1}}{-3+1} + C =$

$= -\frac{\cos x^{-2}}{-2} + C = \frac{1}{2}\frac{1}{\cos^2 x} + C$

Ejercicio 2: Calcular las siguientes integrales indefinidas:

a) $I=\int \frac{\ln^3 x}{x}\,dx$	b) $I=\int (\ln x\cdot sen\,x)^{17}\cdot\left(\frac{sen\,x}{x}+\cos x\cdot\ln x\right)dx$	c) $I=\int 4\text{sen}\,(4x-9)\,dx$
d) $I=\int x\cdot\text{sen}\,(5x^2+3)\,dx$	e) $I=\int (1+tg^2x)\cdot\text{sen}^{-4}x\cdot\cos^4 x\,\,dx$	f) $I=\int \text{sen}\,(7x+3)\,dx$
g) $I=\int (x^2+1)\cdot sen(x^3+3x)\,dx$	h) $I=\int sen\,x\cdot sen(\cos x)\,dx$	j) $I=\int (x-1)\cdot sen(x^2-2x)\,dx$
k) $I=\int e^x\cdot\text{sen}\,(e^x)\,dx$	l) $I=\int \frac{sen(\sqrt{x})}{\sqrt{x}}\,dx$	m) $I=\int \frac{sen(\ln x)}{x}\,dx$
n) $I=\int \cos(4x-3)\,dx$	ñ) $I=\int (x^2+1)\cdot\cos(x^3+3x)\,dx$	o) $I=\int x^2\cdot\cos(x^3+1)\,dx$

2 a) $\int \frac{\ln^3 x}{x}\,dx = \frac{(\ln|x|)^4}{x} + C$

b) $\int (\ln x\cdot \text{sen}\,x)^{17}\cdot\left(\frac{\text{sen}\,x}{x} + \cos x\,\ln x\right)dx = \frac{(\ln x\cdot\text{sen}\,x)^{18}}{18} + C$

c) $\int 4\,\text{sen}(4x-9)\,dx = -\cos(4x-9) + C$

d) $\int x\cdot\text{sen}(5x^2+3)\,dx = -\frac{\cos(5x^2+3)}{10} + C$

e) $\int (1+tg^2x)\cdot\text{sen}^4x\cdot\cos^{4}x\,dx = \int (1+tg^2x)\cdot\left(\frac{\text{sen}\,x}{\cos x}\right)^4 dx =$

$= \frac{(tg\,x)^5}{5} + C$

tg

f) $\int \text{sen}(7x+3)\,dx = -\frac{\cos(7x+3)}{7} + C$

g) $\int (x^2+1)\,\text{sen}(x^3+3x)\,dx = -\frac{\cos(x^3+3x)}{3} + C$

h) $\int \text{sen}\,x\cdot\text{sen}(\cos x)\,dx = -\cos(\cos x) + C$

j) $\int (x-1)\cdot sen(x^2-2x)dx = -\frac{\cos(x^2-2x)}{2} + C$

k) $\int e^x \cdot sen\, e^x dx = -\cos e^x + C$

l) $\int \frac{sen\sqrt{x}}{\sqrt{x}}dx = -2\cos\sqrt{x} + C$

m) $\int \frac{sen(\ln x)}{x}dx = -\cos(\ln x) + C$

n) $\int \cos(4x-3)dx = \frac{sen(4x-3)}{4} + C$

ñ) $\int (x^2+1)\cos(x^3+3x)dx = \frac{sen(x^3+3x)}{3} + C$

o) $\int x^2 \cos(x^3+1)dx = \frac{sen(x^3+1)}{3} + C$

Ejercicio 3: Calcular las siguientes integrales indefinidas:

a) $I=\int \frac{4}{4x-9}\,dx$	b) $I=\int \frac{1}{7x+3}\,dx$	c) $I=\int \frac{x}{5x^2+3}\,dx$
d) $I=\int \frac{x^2+1}{x^3+3x}\,dx$	e) $I=\int \frac{x-1}{x^2-2x}\,dx$	f) $I=\int tgx\,dx$
g) $I=\int \frac{1}{x\cdot\ln x}\,dx$	h) $I=\int \frac{e^x}{1+e^x}\,dx$	j) $I=\int \frac{1}{sen\,x\cdot\cos x}\,dx$
k) $I=\int e^{7x+3}dx$	l) $I=\int x^2\cdot e^{x^3+1}dx$	m) $I=\int (x-1)\cdot e^{x^2-2x}dx$
n) $I=\int \frac{1}{\sqrt{x}}e^{10\sqrt{x}}\,dx$	ñ) $I=\int\left(3x^2e^{x^3+2x}+2e^{x^3+2x}\right)\,dx$	o) $I=\int 5^{7x+3}dx$

3 a) $\int \frac{4}{4x-9}dx = \ln|4x-9| + C$

b) $\int \frac{1}{7x+3}dx = \frac{\ln|7x+3|}{7} + C$

c) $\int \frac{4}{5x^2+3}dx = \int \frac{4/3}{\frac{5}{3}x^2+1}dx = \sqrt{\frac{3}{5}}\,\frac{4}{3}\int \frac{\sqrt{\frac{5}{3}}}{\left(\sqrt{\frac{5}{3}}\,x\right)^2+1}dx = \frac{\sqrt{3}\cdot 4}{\sqrt{5}\cdot 3} \to arctg\sqrt{\frac{5}{3}} + C$

d) $\int \frac{x^2+1}{x^3+3x}dx = \frac{\ln|x^3+3x|}{3} + C$

e) $\int \frac{x-1}{x^2-2x}\,dx = \frac{\ln|x^2-2x|}{2} + C$

f) $\int tg\,x\,dx = \int \frac{sen\,x}{\cos x} = -\ln|\cos x| + C$

g) $\int \frac{1}{x \ln x}\,dx = \int \frac{1}{x}\cdot\frac{1}{\ln x}\cdot dx = \ln(\ln|x|) + C$

h) $\int \frac{e^x}{1+e^x}\,dx = \ln(1+e^x) + C$

i) $\int \frac{1}{sen\,x\cdot\cos x}\,dx = \int \frac{sen^2 x + \cos^2 x}{sen\,x\cdot\cos x}\,dx =$

$= \int \frac{sen^2 x}{sen\,x\cdot\cos x}\,dx + \int \frac{\cos^2 x}{sen\,x\cdot\cos x}\,dx =$

$= \int \frac{sen\,x}{\cos x}\,dx + \int \frac{\cos x}{sen\,x}\,dx =$

$= -\ln|\cos x| + \ln|sen\,x| + C =$

$= \ln|sen\,x| - \ln|\cos x| + C =$

$= \ln\left|\frac{sen\,x}{\cos x}\right| + C = \ln|tg\,x| + C$

k) $\int e^{7x+3}\,dx = \frac{e^{7x+3}}{7} + C$

l) $\int x^2\cdot e^{x^3+1}\,dx = \frac{e^{x^3+1}}{3} + C$

m) $\int (x-1)e^{x^2-2x}\,dx = \frac{e^{x^2-2x}}{2} + C$

n) $\int \frac{1}{\sqrt{x}}\cdot e^{10\sqrt{x}}\,dx = 2\cdot\frac{e^{10\sqrt{x}}}{10} + C$

ñ) $\int \left(3x^2 e^{x^3+2x} + 2e^{x^3+2x}\right)dx = \int e^{x^3+2x}\cdot(3x^2+2)\,dx =$
$= e^{x^3+2x} + C$

o) $\int 5^{7x+3}\,dx = \frac{5^{7x+3}}{7\cdot\ln 5} + C$

Ejercicio 4: Calcular las siguientes integrales indefinidas:

a) $I=\int \frac{1}{x}\cdot 3^{\ln x}\ dx$	b) $I=\int \frac{7^{\sqrt{x}}}{\sqrt{x}}\ dx$	c) $I=\int \frac{2^x}{3^x}\ dx$
d) $I=\int \frac{2}{1+x^2}\ dx$	e) $I=\int \frac{3x^2}{1+x^6}\ dx$	f) $I=\int \frac{1}{3+3x^2}\ dx$
g) $I=\int \frac{x}{1+x^4}\ dx$	h) $I=\int \frac{\cos x}{1+sen^2 x}\ dx$	j) $I=\int \frac{e^x}{1+e^{2x}}\ dx$
k) $I=\int \frac{1}{9+x^2}\ dx$	l) $I=\int x\cdot e^x\ dx$	m) $I=\int x\cdot sen\ x\ dx$
n) $I=\int \ln x\ dx$	ñ) $I=\int arcsen\ x\ dx$	o) $I=\int arctg\ x\ dx$

4 a) $\int \frac{1}{x}\cdot 3^{\ln x} dx = \frac{3^{\ln x}}{\ln 3} + C$

b) $\int \frac{7^{\sqrt{x}}}{\sqrt{x}} dx = 2\cdot \frac{7^{\sqrt{x}}}{\ln 7} + C$

c) $\int \frac{2^x}{3^x} dx = \int \left(\frac{2}{3}\right)^x dx = \left(\frac{2}{3}\right)^x + C$

d) $\int \frac{2}{1+x^2} dx = 2\,arctg\,x + C$

e) $\int \frac{3x^2}{1+x^6} dx = \int \frac{3x^2}{1+(x^3)^2} dx = arc\,tg\,x^3 + C$

f) $\int \frac{1}{3+3x^2} dx = \int \frac{1/3}{1+x^2} dx = \frac{1}{3}\,arctg\,x + C$

g) $\int \frac{x}{1+x^4} dx = \int \frac{x}{1+(x^2)^2} dx = \frac{1}{2}\,arc\,tg\,x^2 + C$

h) $\int \frac{\cos x}{1+sen^2 x} dx = arc\,tg(sen\,x) + C$

j) $\int \frac{e^x}{1+e^{2x}} dx = arc\,tg(e^x) + C$

k) $\int \frac{1}{9+x^2} dx = \int \frac{1/9}{1+\frac{x^2}{9}} dx = 3\cdot\frac{1}{9}\int \frac{\frac{1}{3}}{1+\left(\frac{x}{3}\right)^2} dx = \frac{1}{3}\,arctg\,\frac{x}{3} + C$

l) $\int x\cdot e^x\,dx$

m) $\int x\cdot \operatorname{sen}x\,dx$

n) $\int \ln x\,dx$

ñ) $\int \operatorname{arcsen}x\,dx$

o) $\int \operatorname{arctg}x\,dx$

integración por partes:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$u \to$
- inversas trigonométricas
- logarítmicas
- polinómicas
- trigonométricas
- exponencial

l) $\int x e^x dx = \begin{pmatrix} u=x \longrightarrow du=dx \\ dv=e^x dx \to v=\int e^x dx = e^x \end{pmatrix} =$

$= x\cdot e^x - \int e^x dx = \boxed{x\cdot e^x - e^x + C}$

m) $\int x\cdot \operatorname{sen}x\,dx = \begin{pmatrix} u=x \to du=dx \\ dv=\operatorname{sen}x\,dx \to v=\int \operatorname{sen}x\,dx = -\cos x \end{pmatrix}$

$= -x\cdot\cos x + \int \cos x\,dx = \boxed{-x\cdot\cos x + \operatorname{sen}x + C}$

n) $\int \ln x\,dx = \begin{pmatrix} u=\ln x \to du=\frac{1}{x}dx \\ dv=dx \to v=\int dx = x \end{pmatrix}$

$= x\ln x - \int x\cdot\frac{1}{x}dx = x\ln x - \int dx = \boxed{x\cdot\ln x - x + C}$

ñ) $\int \operatorname{arc}\operatorname{sen}x\,dx = \begin{pmatrix} u=\operatorname{arcsen}x \;;\; du=\frac{1}{\sqrt{1-x^2}}dx \\ dv=dx \;;\; v=\int dx = x \end{pmatrix} \Rightarrow ⊛$

ñ) ⊛ $\Rightarrow$ $u\cdot v - \int v\,du =$

$\int \operatorname{arc}\operatorname{sen}x = x\cdot\operatorname{arcsen}x - \int \frac{2x}{2\sqrt{1-x^2}}dx =$

$= \boxed{x\cdot\operatorname{arc}\operatorname{sen}x + \sqrt{1-x^2} + C}$

o) $\int \operatorname{arc}\operatorname{tg}x\,dx = \begin{pmatrix} u=\operatorname{arctg}x \quad du=\frac{1}{1+x^2}dx \\ dv=dx \quad v=\int dx = x \end{pmatrix}$

$= x\cdot\operatorname{arctg}x - \int \frac{x}{1+x^2}dx =$

$= x\cdot\operatorname{arctg}x - \frac{1}{2}\int \frac{2x}{1+x^2}dx =$

$= \boxed{x\cdot\operatorname{arctg}x - \frac{1}{2}\ln(1+x^2) + C}$

Ejercicio 5: Calcular las siguientes integrales indefinidas:

a) $I=\int x\cdot\ln x\ dx$	b) $I=\int \cos x\cdot\ln\left(sen\ x\right)\ dx$	c) $I=\int \frac{\ln x}{\sqrt{x}}\ dx$
d) $I=\int x^2\cdot\ln x\ dx$	e) $I=\int x\cdot\sqrt{1+x}\ dx$	f) $I=\int\left(2x+2\right)\cdot e^{-2x}\ dx$
g) $I=\int \ln\frac{1}{x}\ dx$	h) $I=\int x^2\cdot\cos x\ dx$	j) $I=\int x^2\cdot e^x\ dx$

a) $\int x\cdot \ln x\, dx = \begin{pmatrix} u=x & du=dx \\ dv=\ln x\, dx & v=\int \ln x\, dx \end{pmatrix}$ NO $\begin{pmatrix} u=\ln x & du=\frac{1}{x}dx \\ dv=x\,dx & ;\ v=\int x\,dx=\frac{x^2}{2} \end{pmatrix}$

$= \frac{x^2}{2}\ln x - \int \frac{1}{x}\cdot\frac{x^2}{2}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\frac{x^2}{2}+C = \boxed{\frac{x^2}{2}\ln x - \frac{x^2}{4}+C}$

b) $\int \cos x\cdot \ln(sen\,x)\,dx \quad \begin{pmatrix} u=\ln(sen\,x) & du=\frac{1}{sen\,x}\cdot\cos x\,dx \\ dv=\cos x\,dx & v=\int\cos x\,dx = sen\,x \end{pmatrix}$

$= sen\,x\cdot\ln(sen\,x) - \int \frac{1}{\cancel{sen\,x}}\cos x\cdot \cancel{sen\,x}$

$= sen\,x\cdot \ln(sen\,x) - \int \cos x\,dx = \boxed{sen\,x\cdot\ln(sen\,x) - sen\,x + C}$

c) $\int \frac{\ln x}{\sqrt{x}}\,dx = \int \ln x\cdot x^{1/2}\,dx = \begin{pmatrix} u=\ln x & du=\frac{1}{x}dx \\ dv=x^{1/2}\,dx & v=\int x^{1/2}dx = \frac{x^{3/2}}{\frac{3}{2}} \end{pmatrix}$

$= \frac{2}{3}\sqrt{x^3}\ln x - \frac{2}{3}\int x^{3/2}\cdot\frac{1}{x}\,dx = \frac{2}{3}\sqrt{x^3}\cdot\ln x - \frac{2}{3}\int x^{1/2}\,dx =$

$= \frac{2}{3}\sqrt{x^3}\cdot\ln x - \frac{2}{3}\cdot\frac{x^{3/2}}{\frac{3}{2}} + C =$

$= \boxed{\frac{2}{3}\sqrt{x^3}\cdot\ln x - \frac{4}{9}\sqrt{x^3}+C}$

d) $\int x^2\cdot \ln x\,dx =$

$\begin{pmatrix} u=\ln x & du=\frac{1}{x}dx \\ dv=x^2dx & ,\ v=\int x^2dx=\frac{x^3}{3} \end{pmatrix}$

$= \frac{x^3}{3}\ln x - \int\frac{x^3}{3}\cdot\frac{1}{x}\,dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx = \boxed{\frac{x^3}{3}\ln x - \frac{1}{9}x^3+C}$

e) $\int x\cdot\sqrt{1+x}\,dx = \begin{pmatrix} u=x & du=dx \\ dv=\sqrt{1+x}\,dx & ;\ v=\int\sqrt{1+x}\,dx = \int(1+x)^{1/2} = \frac{(1+x)^{3/2}}{\frac{3}{2}} \end{pmatrix}$

$= \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\int(1+x)^{3/2}\,dx = \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\frac{(1+x)^{5/2}}{\frac{5}{2}} + C$

$= \frac{2}{3}x\sqrt{(1+x)^3} - \frac{4}{15}\sqrt{(1+x)^5} + C$

f) $\int (2x+2)\cdot e^{-2x}\,dx$ $\quad\left(\begin{array}{ll} u = x+1 & dx = du \\ dv = 2e^{-2x}dx & v = -e^{-2x}\end{array}\right)$

$= -(x+1)e^{-2x} - \int -e^{-2x}\,dx =$

$= -(x+1)e^{-2x} - \frac{1}{2}e^{-2x} + C = e^{-2x}\left(-x-1-\frac{1}{2}\right) + C$

$= -e^{-2x}\cdot\left(+x+\frac{3}{2}\right) + C$

g) $\int \ln\frac{1}{x}\,dx =$ $\quad\left(\begin{array}{ll} u = \ln\frac{1}{x} & du = x\cdot\frac{-1}{x^2} = -\frac{1}{x} \\ dv = dx & v = \int dx = x\end{array}\right)$

$= x\cdot\ln\frac{1}{x} - \int x\cdot\frac{-1}{x}\,dx = x\cdot\ln\frac{1}{x} + x + C$

h) $\int x^2\cos x\,dx =$ $\quad\left(\begin{array}{ll} u = x^2 & du = 2x\,dx \\ dv = \cos x\,dx & v = \int\cos x\,dx = \operatorname{sen} x\end{array}\right) I_1$

$= x^2\cdot\operatorname{sen} x - 2\int x\cdot\operatorname{sen} x\,dx$ $\quad\left(\begin{array}{ll} u = x & du = dx \\ dv = \operatorname{sen} x\,dx & v = -\cos x\end{array}\right) I_2$

$= x^2\cdot\operatorname{sen} x - 2\left(-x\cdot\cos x + \int\cos x\,dx\right) =$

$= x^2\cdot\operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x + C$

j) $\int x^2 e^x\,dx =$ $\quad\left(\begin{array}{ll} u = x^2 & du = 2x\,dx \\ dv = e^x dx & v = \int e^x dx = e^x\end{array}\right) I_1$

$= x^2\cdot e^x - 2\int x\cdot e^x\,dx =$ $\quad\left(\begin{array}{ll} u = x & du = dx \\ dv = e^x dx & v = \int e^x dx = e^x\end{array}\right) I_2$

$= x^2\cdot e^x - 2\left(x\cdot e^x - \int e^x dx\right)$

$= x^2 e^x - 2xe^x + 2e^x + C$

Ejercicio 6: Calcular las siguientes integrales indefinidas:

a) $I=\int \ln^2 x\,dx = \int(\ln x)^2\,dx$	b) $I=\int(4x^2+3x)\cdot\cos x\,dx$	c) $I=\int e^x\cdot sen\,x\,dx$
d) $I=\int sen(\ln x)\,dx$	e) $I=\int\frac{x^2-x+6}{x}\,dx$	f) $I=\int\frac{x^2+1}{x-1}\,dx$
g) $I=\int\frac{x^3}{x^2+1}\,dx$	h) $I=\int\frac{x^2+x+3}{x^2+1}\,dx$	j) $I=\int\frac{x^2+6x-1}{(x+3)^2}\,dx$
k) $I=\int\frac{x}{(x-1)\cdot(x+5)}\,dx$	l) $I=\int\frac{5-x}{(x-1)\cdot(x+4)}\,dx$	m) $I=\int\frac{x+2}{x^2-2x-3}\,dx$

n) $I=\int \frac{-3x-2}{x^2+5x+6}\,dx$	ñ) $I=\int \frac{x}{8x^2-2x-1}\,dx$	o) $I=\int \frac{6}{x^2-1}\,dx$

6) a) $\int \ln^2 x\,dx = \int (\ln x)^2 dx = \quad I_1 \left(\begin{array}{ll} u=(\ln x)^2 & du = 2\ln x \cdot \frac{1}{x} dx \\ dv = dx & v=\int dx = x \end{array} \right)$

$= (\ln x)^2 \cdot x - 2\int \cancel{x} \cdot \ln x \cdot \frac{1}{\cancel{x}}\, dx = (\ln x)^2 \cdot x - 2\int \ln x\, dx$

$= (\ln x)^2 \cdot x - 2\left(x \cdot \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx\right) = \quad I_2 \left(\begin{array}{ll} u=\ln x & du = \frac{1}{x}dx \\ dv=dx & v=x \end{array} \right)$

$= (\ln x)^2 \cdot x - 2x \cdot \ln x + 2x + C$

b) $\int (4x^2+3x)\cos x\, dx = \left(\begin{array}{ll} u=4x^2+3x & du=(8x+3)dx \\ dv=\cos x\, dx & v = \operatorname{sen} x \end{array} \right) I_1$

$= (4x^2+3x)\cdot \operatorname{sen} x - \int (8x+3)\cdot \operatorname{sen} x\, dx \quad I_2 \left(\begin{array}{ll} u=8x+3 & du=8dx \\ dv=\operatorname{sen} x\, dx & v=-\cos x \end{array} \right)$

$= (4x^2+3x)\cdot \operatorname{sen} x - \left((8x+3)\cdot(-\cos x) + \int \cos x \cdot 8\, dx\right) =$

$= (4x^2+3x)\cdot \operatorname{sen} x + (8x+3)\cos x - 8 \cdot \operatorname{sen} x + C$

c) $\int e^x \operatorname{sen} x\, dx = \left(\begin{array}{ll} u=\operatorname{sen} x & du = \cos x\, dx \\ dv = e^x dx & v = \int e^x dx = e^x \end{array} \right) I_1$

$= e^x \operatorname{sen} x - \int e^x \cos x\, dx = \left(\begin{array}{ll} u=\cos x & du = -\operatorname{sen} x\, dx \\ dv = e^x dx & v = e^x \end{array} \right) I_2$

$= e^x \operatorname{sen} x - \left(e^x \cos x + \int e^x \operatorname{sen} x\, dx\right)$

$I = e^x \operatorname{sen} x - e^x \cos x - I; \quad 2I = e^x \operatorname{sen} x - e^x \cos x$

$I = \frac{1}{2} e^x(\operatorname{sen} x - \cos x) + C$

d) $\int \operatorname{sen}(\ln x)\, dx = \left(\begin{array}{ll} u=\operatorname{sen}(\ln x) & du = \cos(\ln x)\cdot \frac{1}{x} dx \\ dv = dx, & v=\int dx = x \end{array} \right)$

$= x \cdot \operatorname{sen}(\ln x) - \int \cancel{x} \cdot \cos(\ln x) \cdot \frac{1}{\cancel{x}}\, dx =$

$= x \cdot \operatorname{sen}(\ln x) - \int \cos(\ln x)\, dx \quad \left(\begin{array}{ll} u=\cos(\ln x) & du = -\operatorname{sen}(\ln x)\frac{1}{x} dx \\ dv = dx; & v=x \end{array} \right)$

$= x \cdot \operatorname{sen}(\ln x) - \left(x \cdot \cos(\ln x) + \int \operatorname{sen}(\ln x) \cdot \frac{1}{\cancel{x}} \cdot \cancel{x}\, dx\right)$

$I = x \cdot \operatorname{sen}(\ln x) - x \cdot \cos(\ln x) - I;$

$$2I = x\,\text{sen}(\ln x) - x\cos(\ln x)$$

$$I = \frac{1}{2}\left(x\cdot\text{sen}(\ln x) - x\cdot\cos(\ln x)\right)$$

e) $\displaystyle\int \frac{x^2-x+6}{x}\,dx = \int\left(x-1+\frac{6}{x}\right)dx = \frac{x^2}{2} - x + 6\ln|x| + C$

f) $\displaystyle\int \frac{x^2+1}{x-1}\,dx = \int(x+1)\,dx + \int\frac{2}{x-1}\,dx = \frac{x^2}{2} + x + 2\ln|x-1| + C$

$$\begin{array}{l|l} x^2 \quad +1 & x-1 \\ \hline -x^2+x & x+1 \\ \quad x+1 & \\ \quad -x+1 & \\ \hline \quad\quad 2 & \end{array}$$

$D = d\cdot c + r$

$\boxed{\dfrac{D}{d} = c + \dfrac{r}{d}}$

$$\begin{array}{l|l} x^3 & x^2+1 \\ \hline -x^3-x & x \\ \hline \quad -x & \end{array}$$

g) $\displaystyle\int \frac{x^3}{x^2+1}\,dx = \int x\,dx - \int\frac{x}{x^2+1}\,dx = \frac{x^2}{2} - \frac{1}{2}\ln(x^2+1) + C$

h) $\displaystyle\int \frac{x^2+x+3}{x^2+1}\,dx = \int 1\,dx + \int\frac{x+3}{x^2+1}\,dx = x + \frac{1}{2}\int\frac{2x}{x^2+1}\,dx + \int\frac{3}{x^2+1}\,dx$

$$= x + \frac{1}{2}\ln(x^2+1) + 3\,\text{arctg}\,x + C$$

$$\begin{array}{l|l} x^2+x+3 & x^2+1 \\ \hline -x^2 \quad\; -1 & 1 \\ \hline \quad\; x+3 & \end{array}$$

$\dfrac{D}{d} = c + \dfrac{r}{d}$

j) $\displaystyle\int \frac{x^2+6x-1}{(x+3)^2}\,dx = \int dx + \int\frac{-10}{(x+3)^2}\,dx = x + \frac{10}{x+3} + C$

$$\begin{array}{l|l} x^2+6x-1 & x^2+6x+9 \\ \hline -x^2-6x-9 & 1 \\ \hline \quad\quad -10 & \end{array}$$

k) $\displaystyle\int \frac{x}{(x+5)(x-1)}\,dx = \int\frac{1/6}{x-1}\,dx + \int\frac{5/6}{x+5}\,dx = \frac{1}{6}\ln|x-1| + \frac{5}{6}\ln|x+5| + C$

$$\frac{x}{(x-1)(x+5)} = \frac{A}{x-1} + \frac{B}{x+5}$$

$x = A(x+5) + B(x-1)$

$x=0 \quad 0 = 5A + B$

$x=1 \quad 1 = 6A \Rightarrow A = 1/6$

$x=-5 \quad -5 = -6B \Rightarrow B = 5/6$

l) $\int \frac{5-x}{(x-1)(x+4)}\,dx = \int \frac{4/5}{x-1}\,dx + \int \frac{-5/9}{x+4}\,dx = \frac{4}{5}\ln|x-1| - \frac{5}{9}\ln|x+4| + C$

$\frac{5-x}{(x-1)(x+4)} = \frac{A}{(x-1)} + \frac{B}{(x+4)}$ $\quad 5-x = A(x+4) + B(x-1)$

$4 = 5A \rightarrow A = 4/5$

$9 = -5B \Rightarrow B = -5/9$

m) $\int \frac{x+2}{x^2-2x-3}\,dx = -\frac{1}{4}\int \frac{dx}{x+1} + \frac{4}{5}\int \frac{dx}{x-3} = -\frac{1}{4}\ln|x+1| + \frac{4}{5}\ln|x-3| + C$

$\frac{x+2}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3}$ $\quad x+2 = A(x-3) + B(x+1)$

$x=3 \quad 5 = 4B \Rightarrow B = 4/5$

$x^2-2x-3 = (x+1)(x-3$ $\quad x=-1 \quad 1 = -4A \Rightarrow A = -1/4$

n) $\int \frac{-3x-2}{x^2+5x+6}\,dx = -7\int \frac{dx}{x+3} + 4\int \frac{dx}{x+2} = -7\ln|x+3| + 4\ln|x+2| + C$

$\frac{-3x-2}{x^2+5x+6}$ $\quad \frac{-3x-2}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$

$-3x-2 = A(x+2) + B(x+3)$

$x=-2 \quad 4 = B \quad B = 4$

$x=-3 \quad 7 = -A \quad ; \quad A = -7$

ñ) $\int \frac{x}{8x^2-2x-1}\,dx = \int \frac{1/12}{x-\frac{1}{2}}\,dx + \int \frac{1/24}{x+\frac{1}{4}}\,dx = \frac{1}{2}\ln|x-\frac{1}{2}| +$

$+ \frac{1}{24}\ln|x+\frac{1}{4}| + C$

$8x^2-2x-1=0 \rightarrow x = 1/2,\ x = -1/4$

$8(x-\frac{1}{2})(x+\frac{1}{4})$

$\frac{x}{8(x-\frac{1}{2})(x+\frac{1}{4})} = \frac{A}{x-\frac{1}{2}} + \frac{B}{x+\frac{1}{4}} + \frac{C}{8}$

$x = 8A(x+\frac{1}{4}) + 8B(x-\frac{1}{2}) + C(x-\frac{1}{2})(x+\frac{1}{4})$

$x = -\frac{1}{4} \quad -\frac{1}{4} = 8B\cdot(-\frac{1}{4}-\frac{1}{2})$

$-\frac{1}{4} = 8B\cdot -\frac{3}{4} \quad ; \quad -1 = -24B \quad ; \quad B = \frac{1}{24}$

$x = \frac{1}{2} \quad \frac{1}{2} = 8A\cdot(\frac{1}{2}+\frac{1}{4})$

$\frac{1}{2} = 8A\cdot\frac{3}{4} \quad ; \quad \cancel{2}\ 1 = 24A \quad ; \quad A = 1/12$

$x=0$

$0 = 2A - 4B + C(-\frac{1}{8})$

$0 = \frac{2}{12} - \frac{4}{24} - \frac{1}{8}C \quad , \quad 0 = 4 - 4 - 3C \quad C = 0$

Otra forma:

$$\int \frac{x}{8x^2-2x-1}\,dx = \int \frac{x}{8\left(x-\frac{1}{2}\right)\left(x+\frac{1}{4}\right)}\,dx = \frac{1}{8}\int \frac{x}{\left(x-\frac{1}{2}\right)\left(x+\frac{1}{4}\right)}\,dx$$

$$= \frac{1}{8}\left(\int \frac{2/3}{x-\frac{1}{2}}\,dx + \int \frac{1/3}{x+\frac{1}{4}}\,dx\right)$$

$$\frac{x}{\left(x-\frac{1}{2}\right)\left(x+\frac{1}{4}\right)} = \frac{A}{x-\frac{1}{2}} + \frac{B}{x+\frac{1}{4}}$$

$$x = A\left(x+\frac{1}{4}\right) + B\left(x-\frac{1}{2}\right)$$

$B = 1/3$ $\quad$ $A = 2/3$

$$= \frac{1}{8}\cdot\frac{2}{3}\ln\left|x-\frac{1}{2}\right| + \frac{1}{8}\cdot\frac{1}{3}\ln\left|x+\frac{1}{4}\right| + C =$$

$$= \frac{1}{12}\cdot\ln\left|x-\frac{1}{2}\right| + \frac{1}{24}\ln\left|x+\frac{1}{4}\right| + C$$

o) $$\int \frac{6}{x^2-1}\,dx = \int \frac{3}{x+1}\,dx + \int \frac{-3}{x-1}\,dx = 3\ln|x+1| - 3\ln|x-1| + C$$

$$x^2-1 = (x+1)(x-1)$$

$$\frac{6}{x^2-1} = \frac{6}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1)+B(x+1)}{(x+1)(x-1)}$$

$$6 = A(x+1) + B(x-1)$$

$x=1 \quad 6 = 2A \Rightarrow A = 6/2 \quad A = 3 \quad A=3$

$x=-1 \quad 6 = -2B \Rightarrow B = -3 \quad B=-3$

Ejercicio 7: Calcular las siguientes integrales indefinidas:

a) $I=\int \frac{7}{4x^2+4x+1}\,dx$	b) $I=\int \frac{x-5}{x^3-x^2-10x-8}\,dx$	c) $I=\int \frac{x+2}{x^3+5x^2+7x+3}\,dx$
d) $I=\int \frac{x+2}{x^3-x^2-x+1}\,dx$	e) $I=\int \frac{\sqrt[3]{x}+\sqrt{5x^3}}{3x}\,dx$	f) $I=\int \frac{x^2-2x+6}{(x-1)^3}\,dx$
g) $I=\int \frac{x^3+22x^2-12x+8}{x^4-4x^2}\,dx$	h) $I=\int (x-sen\,x)\,dx$	j) $I=\int \left(e^x+3e^{-x}\right)dx$
k) $I=\int \left(x^2+4x\right)\cdot\left(x^2-1\right)dx$	l) $I=\int \left(3^x-x^3\right)dx$	m) $I=\int \frac{(1+\ln x)^2}{x}\,dx$

7 a) $$\int \frac{7}{4x^2+4x+1}\,dx = \frac{7}{2}\int \frac{dx}{\left(x+\frac{1}{2}\right)^2} = -\frac{7}{2}\,\frac{\cdot}{x+\frac{1}{2}} + C$$

$$\frac{7}{4x^2+4x+1} = \frac{7}{(2x+1)^2} = \frac{7}{2^2\left(x+\frac{1}{2}\right)\left(x+\frac{1}{2}\right)} = \frac{7}{2}\left(\frac{A}{x+\frac{1}{2}} + \frac{B}{\left(x+\frac{1}{2}\right)^2}\right)$$

$$\frac{1}{\left(x+\frac{1}{2}\right)^2} = \frac{A}{x+\frac{1}{2}} + \frac{B}{\left(x+\frac{1}{2}\right)^2}$$

$$1 = A\left(x+\frac{1}{2}\right) + B$$

$$1 = B$$

$$1 = \frac{1}{2}A + 1 \ ; \quad 0 = \frac{1}{2}A \ ; \quad A = 0$$

b) $$\int \frac{x-5}{x^3-x^2-10x-8}\,dx = \int \frac{5/6}{x+1}\,dx + \int \frac{-7/6}{x+2}\,dx + \int \frac{-1/30}{x-4}\,dx = \circledast$$

$$x^3-x^2-10x-8 = (x+1)(x+2)(x-4)$$

$$\frac{x-5}{x^3-x^2-10x-8} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-4}$$

$$x-5 = A(x+2)(x-4) + B(x+1)(x-4) + C(x+1)(x+2)$$

$x=-2 \quad -7 = B\cdot(-1)\cdot(-6) \ ; \quad B = -7/6$

$x=4 \quad -1 = C\cdot 5\cdot 6 \ ; \quad C = -1/30$

$x=-1 \quad -6 = A\cdot(-5) \ ; \quad A = 5/6$

$$\circledast = \frac{5}{6}\ln|x+1| - \frac{7}{6}\ln|x+2| - \frac{1}{30}\ln|x-4| + C$$

c) $$\int \frac{x+2}{x^3+5x^2+7x+3}\,dx = \int \frac{\frac{1}{4}}{x+1}\,dx + \int \frac{\frac{1}{2}}{(x+1)^2}\,dx + \int \frac{-\frac{1}{4}}{(x+3)}\,dx = \circledast$$

$$x^3+5x^2+7x+3 = (x+1)(x+1)(x+3)$$

$$\frac{x+2}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$$

$$x+2 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$$

$\begin{cases} x=-1 \quad 1 = 2B \ ; \quad B = 1/2 \\ x=-3 \quad -1 = 4C \ ; \quad C = -1/4 \\ x=0 \quad 2 = 3A + 3B + C \ ; \quad 2 = 3A + 3/2 - \frac{1}{4} \end{cases}$

$8 = 12A + 6 - 1 \ ; \quad 3 = 12A \ ; \quad A = 1/4$

$$\circledast = \frac{1}{4}\ln|x+1| + \frac{1}{2}\cdot\frac{-1}{x+1} - \frac{1}{4}\ln|x+3| + C$$

d) $\displaystyle\int \frac{x+2}{x^3-x^2-x+1}\,dx = \int \frac{-1/4}{x-1}\,dx + \int \frac{3/2}{(x-1)^2}\,dx + \int \frac{1/4}{x+1}\,dx = \circledast$

A

$x^3-x^2-x+1 = (x-1)^2\cdot(x+1)$

$\dfrac{x+2}{x^3-x^2-x+1} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1}$

$x+2 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$\begin{cases} x=1 & 3 = 2B \Rightarrow B = 3/2 \\ x=-1 & 1 = 4C \Rightarrow C = 1/4 \\ x=0 & 2 = -A + B + C \end{cases}$; $A = B + C - 2$; $A = \frac{3}{2} + \frac{1}{4} - 2$

$A = -1/4$

$\circledast = -\frac{1}{4}\ln|x-1| - \frac{3}{2}\cdot\frac{1}{(x-1)} + \frac{1}{4}\ln|x+1| + C$

f) $\displaystyle\int \frac{x^3-2x+6}{(x-1)^3}\,dx = \int dx + \int \frac{3x^2-5x+7}{(x-1)^3}\,dx =$

A

$(x-1)^3 = x^3 - 3x^2 + 3x - 1$

$$\begin{array}{l|l} x^3 - 2x + 6 & x^3-3x^2+3x-1 \\ \hline -x^3+3x^2-3x+1 & 1 \\ \hline \quad 3x^2-5x+7 & \end{array}$$

$= x + \int \frac{3}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx + \int \frac{5}{(x-1)^3}$

$= x + 3\ln|x-1| - \frac{1}{x-1} + \frac{5(x-1)^{-2}}{-2}$

$= x + 3\ln|x-1| - \frac{1}{x-1} - \frac{5}{2}\,\frac{1}{(x-1)^2} + C$

$\dfrac{3x^2-5x+7}{(x-1)^3} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3}$

$3x^2-5x+7 = A(x-1)^2 + B(x-1) + C$ $\qquad A=3 \quad B=1$

$x=1 \qquad 5 = C$

$x=0 \qquad 7 = A - B + C$; $\quad \left.\begin{matrix} 7-5 = A-B \\ 15-5 = 4A-2B \end{matrix}\right\}$ $\quad \left.\begin{matrix} 2 = A-B \\ 10 = 4A-2B \end{matrix}\right\}$

$x=-1 \qquad 15 = 4A - 2B + C$

g) $\displaystyle\int \frac{x^3+22x^2-12x+8}{x^4-4x^2}\,dx = \int \frac{-5}{x}\,dx + \int \frac{-2}{x^2}\,dx + \int \frac{-7}{x+2}\,dx + \int \frac{-3}{x-2}\,dx$

$x^4-4x^2 = x^2(x^2-4) = x^2(x+2)(x-2)$

$\dfrac{x^3+22x^2-12x+8}{x^4-4x^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+2} + \dfrac{D}{x-2}$

$x^3+22x^2-12x+8 = Ax(x+2)(x-2) + B(x+2)(x-2) + Cx^2(x-2) + Dx^2(x+2)$

$x=0 \qquad 8 = -4B$; $\quad B = -2$

$x=2 \qquad -48 = 16D$; $\quad D = -3$

$x=-2 \qquad 112 = -16C$; $\quad C = -7$

$x=1 \qquad 19 = -3A - 3B - C + 3D$; $\quad 19 = -3A + 6 + 7 - 9$;

$3A = -19 + 6 + 7 - 9$; $\quad 3A = -15$

$A = -5$

h) $I = \int (x - \operatorname{sen} x)\,dx = \frac{x^2}{2} + \cos x + C$

j) $I = \int (e^x + 3e^{-x})\,dx = e^x - 3\cdot e^{-x} + C$

k) $I = \int (x^2+4x)\cdot(x^2-1)\,dx = \int (x^4+4x^3-x^2-4x)\,dx$

$= \frac{x^5}{5} + \frac{4x^4}{4} - \frac{x^3}{3} - \frac{4x^2}{2} + C = \frac{x^5}{5} + x^4 - \frac{x^3}{3} - 2x^2 + C$

l) $I = \int (3^x - x^3)\,dx = \frac{3^x}{\ln 3} - \frac{x^4}{4} + C$

m) $I = \int \frac{(1+\ln x)^2}{x}\,dx = \frac{(1+\ln x)^3}{3} + C$

(inmediatas todas)

Ejercicio 8: Calcula esta integral, haciendo el cambio $\sqrt{1-x} = t$:

$$\int x^2 \cdot \sqrt{1-x}\; dx$$

(8) $\int x^2 \cdot \sqrt{1-x}\; dx$ con $\boxed{\sqrt{1-x} = t}$

$t^2 = 1-x$

$x = 1-t^2 \quad dx = -2t\,dt$

$I = \int (1-t^2)^2 \cdot t \cdot (-2t)\,dt =$

$= \int (1-2t^2+t^4)\cdot(-2t^2)\,dt =$

$= \int (-2t^2 + 4t^4 - 2t^6)\,dt =$

$= -\frac{2t^3}{3} + \frac{4t^5}{5} - \frac{2t^7}{7} + C =$

$= -\frac{2}{3}\left(\sqrt{1-x}\right)^3 + \frac{4}{5}\left(\sqrt{1-x}\right)^5 - \frac{2}{7}\left(\sqrt{1-x}\right)^7 + C$

Ejercicio 9: Resuelve, utilizando la sustitución $\sqrt{e^x+1} = t$, la siguiente integral:

$$\int \frac{e^{2x}}{\sqrt{e^x+1}}\; dx$$

(9) $\int \frac{e^{2x}}{\sqrt{e^x+1}}\,dx$ con $\boxed{\sqrt{e^x+1}=t}$

$e^x+1=t^2$

$e^x=t^2-1$

$x=\ln(t^2-1)$

$dx=\frac{2t}{t^2-1}\,dt$

$I=\int \frac{(t^2-1)^{\cancel{2}}}{\cancel{t}}\cdot\frac{2\cancel{t}}{\cancel{t^2-1}}\,dt=$

$=\int 2(t^2-1)\,dt$

$=\int 2(t^2-1)\,dt=\frac{2t^3}{3}-2t+C=\frac{2}{3}\left(\sqrt{e^x+1}\right)^3-2\sqrt{e^x+1}+C$

Ejercicio 10: Halla la siguiente integral, haciendo el cambio $x = sen\ t$:

$\int \frac{x^2}{\sqrt{1-x^2}}\,dx$ (Recuerda que $sen^2 x=\frac{1}{2}-\frac{\cos 2x}{2}$)

(10) $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

$x=sen\,t$

$dx=\cos t\,dt$

$\arcsen x=t$

$=\int \frac{sen^2 t}{\sqrt{1-sen^2 t}}\cdot\cos t\,dt=\int \frac{sen^2 t}{\cancel{\cos t}}\cdot\cancel{\cos t}\,dt=$

$=\int sen^2 t\,dt=\circledast$

$=\int\left(\frac{1}{2}-\frac{\cos 2t}{2}\right)dt=$

$=\frac{1}{2}t-\frac{sen\,2t}{4}+C=$

$=\frac{\arcsen x}{2}-\frac{sen\,2\arcsen x}{4}+C$

$\left\{\begin{array}{l}\cos 2x=\cos^2 x-sen^2 x\\ 1=\cos^2 x+sen^2 x\end{array}\right\}$

$\cos 2x+1=2\cos^2 x$

$\boxed{\cos^2 x=\frac{1}{2}+\frac{\cos 2x}{2}}$ Al sumar

$\boxed{sen^2 x=\frac{1}{2}-\frac{\cos 2x}{2}}$ Al restar $\circledast$.

Ejercicio 11: Resuelve por sustitución:

a) $\int x\cdot\sqrt{x+1}\,dx$	b) $\int \frac{dx}{x-\sqrt[4]{x}}$	c) $\int \frac{x}{\sqrt{x+1}}\,dx$

d) $\int \frac{1}{x\sqrt{x+1}}\,dx$	e) $\int \frac{1}{x+\sqrt{x}}\,dx$	f) $\int \frac{\sqrt{x}}{1+x}\,dx$

11 a) $\int x\cdot\sqrt{x+1}\,dx =$ $\quad \boxed{t=\sqrt{x+1}}$ $\quad t^2 = x+1$; $x = t^2-1$; $dx = 2t\,dt$

$= \int (t^2-1)\cdot t\cdot 2t\,dt =$

$= \int (2t^4 - 2t^2)\,dt = \frac{2t^5}{5} - \frac{2t^3}{3} + C =$

$= \frac{2}{5}\left(\sqrt{x+1}\right)^5 - \frac{2}{3}\left(\sqrt{x+1}\right)^3 + C$

b) $\int \frac{dx}{x-\sqrt[4]{x}} =$ $\quad \boxed{t=\sqrt[4]{x}}$ $\quad t^4 = x$; $x = t^4$; $dx = 4t^3\,dt$

$= \int \frac{4t^3}{t^4 - t}\,dt = \int \frac{4t^2}{t^3-1}\,dt$

$$\frac{4t^2}{t^3-1} = \frac{A}{t-1} + \frac{Mx+N}{t^2+t+1}$$

c) $\int \frac{x}{\sqrt{x+1}}\,dx =$ $\quad \boxed{\sqrt{x+1}=t}$ $\quad x+1 = t^2$; $x = t^2-1$; $dx = 2t\,dt$

$= \int \frac{t^2-1}{\cancel{t}}\,2\cancel{t}\,dt = \int 2(t^2-1)\,dt = \frac{2t^3}{3} - 2t + C =$

$= \frac{2\left(\sqrt{1+x}\right)^3}{3} - 2\sqrt{x+1} + C$

d) $\int \frac{1}{x\sqrt{x+1}}\,dx =$ $\quad \boxed{\sqrt{x+1}=t}$ $\quad x+1 = t^2$; $x = t^2-1$; $dx = 2t\,dt$

$= \int \frac{1}{(t^2-1)\cancel{t}}\cdot 2\cancel{t}\,dt =$

$= \int \frac{2}{t^2-1}\,dt = \int \frac{1}{t-1}\,dt - \int \frac{1}{t+1}\,dt =$

$$\frac{2}{t^2-1} = \frac{A}{t-1} + \frac{B}{t+1}$$

$$2 = A(t+1) + B(t-1)$$

$$2 = 2A \Rightarrow A = 1$$

$$2 = -2B \Rightarrow B = -1$$

$$= \ln|t-1| - \ln|t+1| + C$$

$$= \ln\left|\sqrt{x+1}-1\right| - \ln\left|\sqrt{x+1}+1\right|$$

$$= \ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + C$$

e) $\displaystyle\int \frac{1}{x+\sqrt{x}}\,dx =$ $\boxed{\sqrt{x} = t}$ $x = t^2$ $dx = 2t\,dt$

$$= \int \frac{1}{t^2+t}\,2t\,dt =$$

$$= \int \frac{2\cancel{t}}{\cancel{t}(t+1)}\,dt = \int \frac{2}{t+1}\,dt = 2\ln|t+1| + C$$

$$= 2\ln\left|\sqrt{x}+1\right| + C$$

f) $\displaystyle\int \frac{\sqrt{x}}{1+x}\,dx =$ $\boxed{\sqrt{x} = t}$ $x = t^2$ $dx = 2t\,dt$

$$= \int \frac{t}{1+t^2}\cdot 2t\,dt =$$

$$= \int \frac{2t^2}{1+t^2}\,dt = \int 2\,dt - \int \frac{2}{t^2+1}\,dt$$

$$\begin{array}{r|l} 2t^2 & t^2+1 \\ -2t^2-2 & 2 \\ \hline -2 & \end{array}$$

Ejercicio 12: Encuentra la primitiva de $f(x) = \dfrac{1}{1+3x}$ que se anula para $x = 0$.

(12) $f(x) = \dfrac{1}{1+3x}$ $\displaystyle\int \frac{1}{1+3x}\,dx = \frac{1}{3}\int \frac{3}{1+3x}\,dx = \frac{1}{3}\ln|1+3x| + C$

$$F(x) = \frac{1}{3}\ln|1+3x| + C$$

$$F(0) = 0 \Rightarrow \frac{1}{3}\ln 1 + C = 0$$

$$C = 0$$

$$\boxed{F(x) = \frac{1}{3}\ln|1+3x|}$$

Ejercicio 13: De todas las primitivas de la función $y = 4x-6$, ¿cuál de ellas toma el valor de 4 para $x = 1$.

(13) $\displaystyle\int (4x-6)\,dx = \frac{4x^2}{2} - 6x + C = 2x^2 - 6x + C$

$$F(x) = 2x^2 - 6x + C$$

$$F(1) = 4 \Rightarrow 2 - 6 + C = 4; \quad -4 + C = 4$$

$$C = 8$$

$$\boxed{F(x) = 2x^2 - 6x + 8}$$

Ejercicio 14: Halla $f(x)$ sabiendo que $f''(x) = 6x$, $f'(0) = 1$ y $f(2) = 5$.

(14) $f(x)$ si $f''(x) = 6x$, $f'(0) = 1$ y $f(2) = 5$

$f'(x) = \int 6x\,dx = \frac{6x^2}{2} + C$

$f'(x) = 3x^2 + C$ con $f'(0) = 1 \Rightarrow +C = 1$; $C = 1$

$\boxed{f'(x) = 3x^2 + 1}$

$f(x) = \int (3x^2+1)\,dx = \frac{3x^3}{3} + x + C$

si $f(2) = 5 \Rightarrow 8 + 2 + C = 5$; $C = -5$

$\boxed{f(x) = x^3 + x - 5}$

Ejercicio 15: Resuelve las siguientes integrales por sustitución:

a) $\int \frac{e^x}{1-\sqrt{e^x}}\,dx$ b) $\int \sqrt{e^x - 1}\,dx$

(15) a) $\int \frac{e^x}{1-\sqrt{e^x}}\,dx =$

$\boxed{\sqrt{e^x} = t}$

$e^x = t^2$

$x = \ln t^2$

$dx = \frac{2t}{t^2}\,dt$; $dx = \frac{2}{t}\,dt$

$2t \;\big|\; -t+1$

$-2t+2 \;\; -2$

$+2$

$= \int \frac{t^2}{1-t} \cdot \frac{2}{t}\,dt$

$= \int \frac{2t}{1-t}\,dt = -2\ln|1-t| - 2t + C =$

$= \int -2\,dt + \int \frac{2}{-t+1}\,dt$

$= -2\ln\left|1-\sqrt{e^x}\right| - 2\sqrt{e^x} + C$

b) $\int \sqrt{e^x - 1}\,dx =$

$\boxed{\sqrt{e^x - 1} = t}$

$e^x - 1 = t^2$

$e^x = t^2 + 1$

$x = \ln(t^2+1)$

$dx = \frac{2t}{t^2+1}\,dt$

$= \int t \cdot \frac{2t}{t^2+1}\,dt =$

$= \int \frac{2t^2}{t^2+1}\,dt =$

$= \int 2\,dt - \int \frac{2}{t^2+1}\,dt = \int 2\,dt - \int \frac{2}{1+t^2}\,dt =$

$= 2t - 2\,\mathrm{arctg}\,t + C = 2\sqrt{e^x - 1} - 2\,\mathrm{arctg}\sqrt{e^x - 1} + C$

Ejercicio 16: Determina la función $f(x)$ sabiendo que $f''(x) = x \cdot \ln x$, $f'(1) = 0$ y $f(e) = \dfrac{e}{4}$.

(16) $f''(x) = x \cdot \ln x$, $f'(1) = 0$ y $f(e) = \dfrac{e}{4}$. Calcula $f(x)$!

$$f'(x) = \int x \cdot \ln x \, dx \qquad \left(\begin{array}{ll} u = \ln x & du = \frac{1}{x} dx \\ dv = x\, dx & v = \int x\, dx = \frac{x^2}{2} \end{array} \right)$$

$$f'(x) = \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx =$$

$$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx = \ln x \cdot \frac{x^2}{2} - \frac{x^2}{4} + C \ ; \quad f'(x) = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$f'(1) = 0 \Rightarrow \ln 1 \cdot \frac{1}{2} - \frac{1}{4} + C = 0 \ ; \ \boxed{C = \frac{1}{4}}$$

$$\boxed{f'(x) = \frac{x^2}{2} \cdot \ln x - \frac{x^2}{4} + \frac{1}{4}}$$

$$f(x) = \int \left(\frac{x^2}{2} \ln x - \frac{x^2}{4} + \frac{1}{4} \right) dx = \frac{1}{2} \underbrace{\int x^2 \cdot \ln x \, dx}_{(*)} - \frac{1}{4} \int x^2 dx + \frac{1}{4} \int dx$$

$$(*) \int x^2 \cdot \ln x \, dx \qquad \left(\begin{array}{ll} u = \ln x & du = \frac{1}{x} dx \\ dv = x^2 \cdot dx & v = \int x^2 dx = \frac{x^3}{3} \end{array} \right)$$

$$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx =$$

$$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 dx = \ln x \cdot \frac{x^3}{3} - \frac{x^3}{9} \ (*)$$

$$f(x) = \frac{1}{2} \left(\ln x \cdot \frac{x^3}{3} - \frac{x^3}{9} \right) - \frac{1}{4} \cdot \frac{x^3}{3} + \frac{1}{4} x + C$$

$$\text{Si } f(e) = \frac{e}{4} \Rightarrow \frac{1}{2} \left(\ln e \cdot \frac{e^3}{3} - \frac{e^3}{9} \right) - \frac{e^3}{12} + \frac{e}{4} + C = \frac{e}{4}$$

$$\frac{1}{2} \left(\frac{e^3}{3} - \frac{e^3}{9} \right) + \cancel{\frac{e}{4}} - \frac{e^3}{12} - \cancel{\frac{e}{4}} = -C$$

$$\frac{e^3}{6} - \frac{e^3}{18} - \frac{e^3}{12} = -C$$

$$\frac{6e^3 - 2e^3 - 3e^3}{36} = -C$$

$$+\frac{e^3}{36} = -C$$

$$C = -\frac{e^3}{36}$$

$$C = \frac{9e - 5e^3}{36}$$

$$\boxed{f(x) = \ln x \cdot \frac{x^3}{6} - \frac{5x^3}{18} - \frac{e^3}{36}}$$

$$f(x) = \frac{\ln x \cdot x^3}{6} - \frac{x^3}{18} - \frac{x^3}{12} + \frac{1}{4} x + C$$

Ejercicio 17: Encuentra la función derivable $f : [-1,1] \to \mathbb{R}$ que cumple que $f(1) = -1$ y

$$f'(x) = \begin{cases} x^2 - 2x & si \quad -1 \le x < 0 \\ e^x - 1 & si \quad 0 \le x \le 1 \end{cases}$$

17) $f(1)=-1$ y $f'(x)=\begin{cases} x^2-2x & si \quad -1\le x<0 \\ e^x-1 & si \quad 0\le x\le 1 \end{cases}$

· $\int (x^2-2x)\,dx = \frac{x^3}{3}-\frac{2x^2}{2}+C = \frac{x^3}{3}-x^2+C$

· $\int (e^x-1)\,dx = e^x-x+C'$

$f(1)=-1$

$e-1+C'=-1$

$C'=-e$

$f(x)=\begin{cases} \frac{x^3}{3}-x^2+C & -1\le x<0 \\ e^x-x-e & 0\le x\le 1 \end{cases}$

Debe ser continua en $x=0$

$f^-(0)=C \qquad f^+(0)=+1-e$

Entonces: $\boxed{f(x)=\begin{cases} \frac{x^3}{3}-x^2+1-e & -1\le x<0 \\ e^x-x-e & 0\le x<1 \end{cases}}$

Ejercicio 18: De una función derivable se sabe que pasa por el punto $A(-1,-4)$ y que su derivada es:

$$f'(x)=\begin{cases} 2-x & si \quad x\le 1 \\ \frac{1}{x} & si \quad x>1 \end{cases}$$

a) Halla la expresión de $f(x)$.
b) Obtén la ecuación de la recta tangente a $f(x)$ en $x=2$.

18) $A(-1,-4)$ y $f'(x)=\begin{cases} 2-x & si \quad x\le 1 \\ \frac{1}{x} & si \quad x>1 \end{cases}$

a) $f(x)=\begin{cases} \int (2-x)\,dx = 2x-\frac{x^2}{2}+C_1 & x\le 1 \\ \int \frac{1}{x}\,dx = \ln x+C_2 & x>1 \end{cases}$

Si $f(-1)=-4 \Rightarrow 2\cdot(-1)-\frac{1}{2}+C_1=-4$

$-2-\frac{1}{2}+4=-C_1$

$\frac{-4-1+8}{2}=-C_1$

$C_1=-\frac{3}{2}$

$f(x)=\begin{cases} 2x-\frac{x^2}{2}-\frac{3}{2} & x\le 1 \\ \ln x+C_2 & x>1 \end{cases}$

Debe ser continua en $x=1$

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1} 2x-\frac{x^2}{2}-\frac{3}{2} = 2-\frac{1}{2}-\frac{3}{2}=\frac{4-1-3}{2}=0$

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1} \ln x+C_2=\ln 1+C_2=C_2$

$C_2=0 \Rightarrow C_2=0$

Así: $\boxed{f(x)=\begin{cases} 2x-\frac{x^2}{2}-\frac{3}{2} & x\le 1 \\ \ln x & x>1 \end{cases}}$

Ejercicio 19: De la función $f:\mathbb{R}\to\mathbb{R}$ se sabe que $f''(x)=x^2+2x+2$ y que su gráfica tiene tangente horizontal en el punto $P(1,2)$. Halla la expresión de f.

19) f tal que $f''(x)=x^2+2x+2$ y su gráfica tiene tangente horizontal en $P(1,2)$

$$f'(x)=\int (x^2+2x+2)\,dx = \frac{x^3}{3}+\frac{2x^2}{2}+2x+C$$

$f'(1)=0 \Rightarrow \frac{1}{3}+1+2+C=0$

(tg horizontal en $x=1$) $\quad \frac{10}{3}+C=0$; $C=-\frac{10}{3}$

$$f(x)=\int\left(\frac{x^3}{3}+\frac{2x^2}{2}+2x-\frac{10}{3}\right)dx=\frac{x^4}{12}+\frac{x^3}{3}+x^2-\frac{10}{3}x+C'$$

$f(1)=2 \Rightarrow \frac{1}{12}+\frac{1}{3}+1-\frac{10}{3}+C'=2$

$$C'=2-\frac{1}{12}-\frac{1}{3}-1+\frac{10}{3}=\frac{24-1-4-12+40}{12}$$

$C'=\frac{47}{12}$ $\quad$ Así: $\boxed{f(x)=\frac{x^4}{12}+\frac{x^3}{3}+x^2-\frac{10}{3}x+\frac{47}{12}}$

Ejercicio 20: Sea $I=\int\frac{5}{1+\sqrt{e^{-x}}}dx$

a) Expresa I haciendo el cambio de variable $t^2=e^{-x}$.
b) Determina I.

20) $I=\int\frac{5}{1+\sqrt{e^{-x}}}dx$

a) $\boxed{t^2=e^{-x}}$ $\quad \ln t^2=-x$; $x=-\ln t^2$

$t=\sqrt{e^{-x}}$ $\quad dx=-\frac{2t}{t^2}dt$

$dx=-\frac{2}{t}dt$

$$I=\int\frac{5}{1+t}\cdot-\frac{2}{t}dt$$

$$I=\int\frac{-10}{t(1+t)}dt$$

$\frac{-10}{t(1+t)}=\frac{A}{t}+\frac{B}{1+t}$

b) $I=\int\frac{-10}{t}dt+\int\frac{10}{t+1}dt$

$-10=A(1+t)+Bt$

$-10=A$

$-10=-B$; $B=10$