

EJERCICIOS DE RADICALES

Ejercicio 1:

Transforma los radicales en potencias, y viceversa.

a) $3^{\frac{1}{4}}$

d) $7^{\frac{3}{5}}$

b) $5^{\frac{2}{3}}$

e) $10^{\frac{2}{7}}$

c) $2^{\frac{1}{6}}$

f) $\sqrt[4]{5^7}$

a) $3^{\frac{1}{4}} = \sqrt[4]{3}$

d) $7^{\frac{3}{5}} = \sqrt[5]{7^3}$

b) $5^{\frac{2}{3}} = \sqrt[3]{5^2}$

e) $10^{\frac{2}{7}} = \sqrt[7]{10^2}$

c) $2^{\frac{1}{6}} = \sqrt[6]{2}$

f) $\sqrt[4]{5^7} = 5^{\frac{7}{4}}$

Ejercicio 2:

Indica si son equivalentes los siguientes radicales.

a) $\sqrt[4]{3^6}$ y $\sqrt{3^3}$

c) $\sqrt[4]{36}$ y $\sqrt{6}$

b) $\sqrt[5]{2^{10}}$ y $\sqrt{2}$

d) $\sqrt[4]{5^{10}}$ y $\sqrt{5^4}$

a) Son equivalentes.

c) Son equivalentes.

b) No son equivalentes.

d) No son equivalentes.

Ejercicio 3:

Efectúa estas operaciones.

a) $\sqrt{20} - 3\sqrt{125} + 2\sqrt{45}$

b) $7\sqrt[3]{81} - 2\sqrt[6]{3^2} + \frac{\sqrt[3]{3}}{5}$

a) $\sqrt{20} - 3\sqrt{125} + 2\sqrt{45} = 2\sqrt{5} - 15\sqrt{5} + 6\sqrt{5} = -7\sqrt{5}$

b) $7\sqrt[3]{81} - 2\sqrt[6]{3^2} + \frac{\sqrt[3]{3}}{5} = 21\sqrt[3]{3} - 2\sqrt[3]{3} + \frac{\sqrt[3]{3}}{5} = \frac{96\sqrt[3]{3}}{5}$

Ejercicio 4:

Opera y simplifica.

a) $4\sqrt{27} \cdot 5\sqrt{6}$

c) $\sqrt[3]{2} \cdot \sqrt{3}$

b) $\left(\frac{\sqrt[6]{32}}{\sqrt{8}}\right)^3$

d) $\frac{\sqrt{3} \cdot \sqrt[3]{3}}{\sqrt[4]{3}}$

a) $4\sqrt{27} \cdot 5\sqrt{6} = 20\sqrt{162} = 180\sqrt{2}$

b) $\left(\frac{\sqrt[6]{32}}{\sqrt{8}}\right)^3 = \frac{\sqrt{32}}{\sqrt{8^3}} = \sqrt{\frac{2^5}{2^9}} = \frac{1}{4}$

c) $\sqrt[3]{2} \cdot \sqrt{3} = \sqrt[6]{4} \cdot \sqrt[6]{27} = \sqrt[6]{108}$

d) $\frac{\sqrt{3} \cdot \sqrt[3]{3}}{\sqrt[4]{3}} = \sqrt[12]{\frac{3^6 \cdot 3^4}{3^3}} = \sqrt[12]{3^7}$

Ejercicio 5:

Racionaliza las siguientes expresiones.

$$\text{a) } \frac{2}{\sqrt{5}} \qquad \text{b) } \frac{-3}{5\sqrt[4]{2^3}} \qquad \text{c) } \frac{2+\sqrt{3}}{6\sqrt[5]{7^3}}$$

$$\text{a) } \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\text{b) } \frac{-3}{5\sqrt[4]{2^3}} = \frac{-3\sqrt[4]{2}}{10}$$

$$\text{c) } \frac{2+\sqrt{3}}{6\sqrt[5]{7^3}} = \frac{(2+\sqrt{3})\sqrt[5]{7^2}}{42}$$

Ejercicio 6:

Racionaliza y opera.

$$\text{a) } \frac{3}{\sqrt{5}} + \frac{4}{\sqrt{6}} \qquad \text{b) } \frac{-7}{3\sqrt{2}} + \frac{5}{4\sqrt{7}}$$

$$\text{a) } \frac{3}{\sqrt{5}} + \frac{4}{\sqrt{6}} = \frac{3\sqrt{5}}{5} + \frac{4\sqrt{6}}{6} = \frac{18\sqrt{5} + 20\sqrt{6}}{30}$$

$$\text{b) } \frac{-7}{3\sqrt{2}} + \frac{5}{4\sqrt{7}} = \frac{-7\sqrt{2}}{6} + \frac{5\sqrt{7}}{28} = \frac{-98\sqrt{2} + 15\sqrt{7}}{84}$$

Ejercicio 7:

Racionaliza y opera.

$$\text{a) } \frac{1}{1+\sqrt{2}} \qquad \text{b) } \frac{8\sqrt{2}}{\sqrt{3}+7} \qquad \text{c) } \frac{5\sqrt{3}}{9-\sqrt{5}}$$

$$\text{a) } \frac{1}{1+\sqrt{2}} = \frac{1-\sqrt{2}}{-1} = -1+\sqrt{2}$$

$$\text{b) } \frac{8\sqrt{2}}{\sqrt{3}+7} = \frac{8\sqrt{6}-56\sqrt{2}}{-46} = \frac{-4\sqrt{6}+28\sqrt{2}}{23}$$

$$\text{c) } \frac{5\sqrt{3}}{9-\sqrt{5}} = \frac{45\sqrt{3}+5\sqrt{15}}{76}$$

Ejercicio 8:

Escribe como potencias de exponente fraccionario estos radicales.

$$\begin{array}{llll} \text{a) } \sqrt{a\sqrt{a}} & \text{c) } \sqrt{\frac{a}{\sqrt{a}}} & \text{e) } \frac{1}{\sqrt{a}} & \text{g) } (\sqrt{a})^3 \\ \text{b) } \sqrt[3]{a\sqrt{a\sqrt{a}}} & \text{d) } \sqrt[4]{a^{-5}} & \text{f) } \frac{1}{\sqrt[4]{a}} & \text{h) } \sqrt[3]{\frac{1}{a}} \end{array}$$

$$\text{a) } \sqrt{a\sqrt{a}} = \left(a \cdot a^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(a^{\frac{3}{2}}\right)^{\frac{1}{2}} = a^{\frac{3}{4}}$$

$$\text{b) } \sqrt[3]{a\sqrt{a\sqrt{a}}} = \left(a \left(a \cdot a^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(a \left(a^{\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(a \cdot a^{\frac{3}{4}}\right)^{\frac{1}{3}} = \left(a^{\frac{7}{4}}\right)^{\frac{1}{3}} = a^{\frac{7}{12}}$$

$$\text{c) } \sqrt{\frac{a}{\sqrt{a}}} = \left(\frac{a}{a^{\frac{1}{2}}}\right)^{\frac{1}{2}} = \left(a^{\frac{1}{2}}\right)^{\frac{1}{2}} = a^{\frac{1}{4}}$$

$$\text{f) } \frac{1}{\sqrt[4]{a}} = \frac{1}{a^{\frac{1}{4}}} = a^{-\frac{1}{4}}$$

$$\text{d) } \sqrt[4]{a^{-5}} = a^{-\frac{5}{4}}$$

$$\text{g) } (\sqrt{a})^3 = a^{\frac{3}{2}}$$

$$\text{e) } \frac{1}{\sqrt{a}} = \frac{1}{a^{\frac{1}{2}}} = a^{-\frac{1}{2}}$$

$$\text{h) } \sqrt[3]{\frac{1}{a}} = a^{-\frac{1}{3}}$$

Ejercicio 9:

Expresa mediante un solo radical.

$$\text{a) } \sqrt[5]{3\sqrt{5}} \quad \text{b) } \sqrt{\frac{\sqrt{2}}{\sqrt[3]{2}}} \quad \text{c) } \sqrt{\sqrt{\sqrt{3}}} \quad \text{d) } \sqrt{\frac{1}{\sqrt{2}}} \quad \text{e) } \sqrt[3]{\sqrt[4]{2}}$$

$$\text{a) } \sqrt[5]{3\sqrt{5}} = \left(3 \cdot 5^{\frac{1}{2}}\right)^{\frac{1}{5}} = 3^{\frac{1}{5}} \cdot 5^{\frac{1}{10}} = 3^{\frac{2}{10}} \cdot 5^{\frac{1}{10}} = \sqrt[10]{3^2 \cdot 5}$$

$$\text{b) } \sqrt{\frac{\sqrt{2}}{\sqrt[3]{2}}} = \left(\frac{2^{\frac{1}{2}}}{2^{\frac{1}{3}}}\right)^{\frac{1}{2}} = \left(2^{\frac{1}{6}}\right)^{\frac{1}{2}} = 2^{\frac{1}{12}} = \sqrt[12]{2}$$

$$\text{c) } \sqrt{\sqrt{\sqrt{3}}} = \left(\left(3^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = 3^{\frac{1}{8}} = \sqrt[8]{3}$$

$$\text{d) } \sqrt{\frac{1}{\sqrt{2}}} = \left(\frac{1}{2^{\frac{1}{2}}}\right)^{\frac{1}{2}} = \left(2^{-\frac{1}{2}}\right)^{\frac{1}{2}} = 2^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{2}}$$

$$\text{e) } \sqrt[3]{\sqrt[4]{2}} = \left(2^{\frac{1}{4}}\right)^{\frac{1}{3}} = 2^{\frac{1}{12}} = \sqrt[12]{2}$$

$$\text{f) } \frac{1}{\sqrt{\sqrt{5}}} = \frac{1}{\left(5^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \frac{1}{5^{\frac{1}{4}}} = 5^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{5}}$$

Ejercicio 10:

Extrae los factores que puedas de la raíz.

- a) $\sqrt{8}$ c) $\sqrt{50}$ e) $\sqrt{12}$ g) $\sqrt[3]{1.000}$
 b) $\sqrt{18}$ d) $\sqrt{98}$ f) $\sqrt{75}$ h) $\sqrt[3]{40}$

$$a) \sqrt{8} = \sqrt{2^3} = 2\sqrt{2}$$

$$b) \sqrt{18} = \sqrt{2 \cdot 3^2} = 3\sqrt{2}$$

$$c) \sqrt{50} = \sqrt{2 \cdot 5^2} = 5\sqrt{2}$$

$$d) \sqrt{98} = \sqrt{2 \cdot 7^2} = 7\sqrt{2}$$

$$e) \sqrt{12} = \sqrt{3 \cdot 2^2} = 2\sqrt{3}$$

$$f) \sqrt{75} = \sqrt{3 \cdot 5^2} = 5\sqrt{3}$$

$$g) \sqrt[3]{1.000} = \sqrt[3]{2^3 \cdot 5^3} = 2 \cdot 5 = 10$$

$$h) \sqrt[3]{40} = \sqrt[3]{2^3 \cdot 5} = 2\sqrt[3]{5}$$

Ejercicio 11:

Extrae factores de los radicales.

- a) $\sqrt[3]{8a^5}$ c) $\sqrt{2^6 a^4 b^8}$ e) $\sqrt[5]{a^6 b^{10}}$
 b) $\sqrt[4]{16a^7}$ d) $\sqrt[4]{a^6 b^5 c^9}$ f) $\sqrt[3]{15.625x^4 y^3}$

$$a) \sqrt[3]{8a^5} = \sqrt[3]{2^3 a^5} = 2a\sqrt[3]{a^2}$$

$$b) \sqrt[4]{16a^7} = \sqrt[4]{2^4 a^7} = 2a\sqrt[4]{a^3}$$

$$c) \sqrt{2^6 a^4 b^8} = 2^3 a^2 b^4$$

$$d) \sqrt[4]{a^6 b^5 c^9} = abc^2 \sqrt[4]{a^2 bc}$$

$$e) \sqrt[5]{a^6 b^{10}} = ab^2 \sqrt[5]{a}$$

$$f) \sqrt[3]{15.625x^4 y^3} = \sqrt[3]{5^6 x^4 y^3} = 5^2 xy \sqrt[3]{x}$$

Ejercicio 12:

Introduce los factores bajo el radical.

- a) $2\sqrt[3]{5}$ c) $3\sqrt[5]{15}$ e) $\frac{1}{2}\sqrt[4]{6}$ g) $2\sqrt[3]{7}$ i) $\frac{3}{5}\sqrt[3]{\frac{2}{3}}$
 b) $4\sqrt[4]{20}$ d) $\frac{3}{5}\sqrt{2}$ f) $\frac{1}{2}\sqrt[4]{\frac{1}{2}}$ h) $5\sqrt[3]{\frac{1}{5}}$ j) $\frac{1}{7} \cdot \frac{\sqrt[3]{3}}{4}$

$$a) 2\sqrt[3]{5} = \sqrt[3]{2^3 \cdot 5} = \sqrt[3]{40}$$

$$b) 4\sqrt[4]{20} = \sqrt[4]{4^4 \cdot 20} = \sqrt[4]{5.120}$$

$$c) 3\sqrt[5]{15} = \sqrt[5]{3^5 \cdot 15} = \sqrt[5]{3.645}$$

$$d) \frac{3}{5}\sqrt{2} = \sqrt{\frac{3^2 \cdot 2}{5^2}} = \sqrt{\frac{18}{25}}$$

$$e) \frac{1}{2}\sqrt[4]{6} = \sqrt[4]{\frac{1 \cdot 6}{2^4}} = \sqrt[4]{\frac{6}{16}} = \sqrt[4]{\frac{3}{8}}$$

$$f) \frac{1}{2}\sqrt[4]{\frac{1}{2}} = \sqrt[4]{\frac{1}{2^4 \cdot 2}} = \sqrt[4]{\frac{1}{32}}$$

$$g) 2\sqrt[3]{7} = \sqrt[3]{2^3 \cdot 7} = \sqrt[3]{56}$$

$$h) 5\sqrt[3]{\frac{1}{5}} = \sqrt[3]{\frac{5^3}{5}} = \sqrt[3]{5^2} = \sqrt[3]{25}$$

$$i) \frac{3}{5}\sqrt[3]{\frac{2}{3}} = \sqrt[3]{\frac{3^3 \cdot 2}{5^3 \cdot 3}} = \sqrt[3]{\frac{18}{125}}$$

$$j) \frac{1}{7} \cdot \frac{\sqrt[3]{3}}{4} = \sqrt[3]{\frac{3}{7^3 \cdot 4^3}} = \sqrt[3]{\frac{3}{21.952}}$$

Ejercicio 13:

Introduce los factores dentro del radical, si es posible.

$$\text{a) } a \cdot \sqrt{\frac{4a-1}{2a}} \quad \text{c) } \frac{2}{a} \cdot \sqrt{\frac{3a}{8}} \quad \text{e) } 5 + \sqrt{2}$$

$$\text{b) } \frac{4ab}{c} \cdot \sqrt[4]{\frac{c^2b}{8a}} \quad \text{d) } -2ab^2 \sqrt[3]{ab} \quad \text{f) } -a^2 \sqrt[3]{a}$$

$$\text{a) } a \cdot \sqrt{\frac{4a-1}{2a}} = \sqrt{\frac{a^2(4a-1)}{2a}} = \sqrt{\frac{4a^2-a}{2}}$$

$$\text{b) } \frac{4ab}{c} \cdot \sqrt[4]{\frac{c^2b}{8a}} = \sqrt[4]{\frac{4^4 a^4 b^4 c^2 b}{c^4 8a}} = \sqrt[4]{\frac{2^8 a^4 b^5 c^2}{2^3 a c^4}} = \sqrt[4]{\frac{2^5 a^3 b^5}{c^2}}$$

$$\text{c) } \frac{2}{a} \cdot \sqrt{\frac{3a}{8}} = \sqrt{\frac{2^2 3a}{2^3 a^2}} = \sqrt{\frac{3}{2a}}$$

$$\text{d) } -2ab^2 \sqrt[3]{ab} = \sqrt[3]{-2^3 a^3 b^6 ab} = \sqrt[3]{-2^3 a^4 b^7}$$

e) No es posible introducir factores, puesto que 5 no es factor.

$$\text{f) } -a^2 \sqrt[3]{a} = \sqrt[3]{-a^6 a} = \sqrt[3]{-a^7}$$

Ejercicio 14:

Opera y simplifica.

$$\begin{array}{ll} \text{a) } (3\sqrt{2} - 5) \cdot (4\sqrt{2} - 3) & \text{e) } (7\sqrt{5} + 4) \cdot (5\sqrt{5} - 3\sqrt{6}) \\ \text{b) } (2\sqrt{7} + 3\sqrt{2}) \cdot (5 - 2\sqrt{2}) & \text{f) } (7\sqrt{2} - 3) \cdot (5\sqrt{3} + 2) \\ \text{c) } (\sqrt{3} + \sqrt{2}) \cdot (\sqrt{3} - \sqrt{2}) & \text{g) } (6\sqrt{7} + \sqrt{5}) \cdot (6\sqrt{7} - \sqrt{5}) \\ \text{d) } (5\sqrt{2} - 3) \cdot (5\sqrt{2} + 3) & \text{h) } (2\sqrt{5} - \sqrt{10}) \cdot (2\sqrt{5} + \sqrt{10}) \end{array}$$

$$\text{a) } (3\sqrt{2} - 5) \cdot (4\sqrt{2} - 3) = 12(\sqrt{2})^2 - 9\sqrt{2} - 20\sqrt{2} + 15 = -29\sqrt{2} + 39$$

$$\text{b) } (2\sqrt{7} + 3\sqrt{2}) \cdot (5 - 2\sqrt{2}) = 10\sqrt{7} - 4\sqrt{14} + 15\sqrt{2} - 6(\sqrt{2})^2 = \\ = 10\sqrt{7} - 4\sqrt{14} + 15\sqrt{2} - 12$$

$$\text{c) } (\sqrt{3} + \sqrt{2}) \cdot (\sqrt{3} - \sqrt{2}) = (\sqrt{3})^2 - \sqrt{6} + \sqrt{6} - (\sqrt{2})^2 = 3 - 2 = 1$$

$$\text{d) } (5\sqrt{2} - 3) \cdot (5\sqrt{2} + 3) = 25(\sqrt{2})^2 + 15\sqrt{2} - 15\sqrt{2} - 9 = 50 - 9 = 41$$

$$\text{e) } (7\sqrt{5} + 4) \cdot (5\sqrt{5} - 3\sqrt{6}) = 35(\sqrt{5})^2 - 21\sqrt{30} + 20\sqrt{5} - 12\sqrt{6} = \\ = 175 - 21\sqrt{30} + 20\sqrt{5} - 12\sqrt{6}$$

$$\text{f) } (7\sqrt{2} - 3) \cdot (5\sqrt{3} + 2) = 35\sqrt{6} + 14\sqrt{2} - 15\sqrt{3} - 6$$

$$\text{g) } (6\sqrt{7} + \sqrt{5}) \cdot (6\sqrt{7} - \sqrt{5}) = 36(\sqrt{7})^2 - 6\sqrt{35} + 6\sqrt{35} - (\sqrt{5})^2 = \\ = 252 - 5 = 247$$

$$\text{h) } (2\sqrt{5} - \sqrt{10}) \cdot (2\sqrt{5} + \sqrt{10}) = 4(\sqrt{5})^2 + 2\sqrt{50} - 2\sqrt{50} - (\sqrt{10})^2 = \\ = 20 - 10 = 10$$

Ejercicio 15:

Calcula.

a) $\sqrt[4]{a^3} \cdot \sqrt[3]{a^5} \cdot \sqrt[6]{a^4}$

c) $\sqrt[5]{2a^3b^4} : \sqrt[3]{4ab^2}$

b) $\sqrt[3]{3a^2b} \cdot \sqrt{2ab^3}$

d) $\sqrt[3]{\sqrt{ab}} \cdot \sqrt{a\sqrt[3]{b}}$

a) $\sqrt[4]{a^3} \cdot \sqrt[3]{a^5} \cdot \sqrt[6]{a^4} = a^{\frac{3}{4}} \cdot a^{\frac{5}{3}} \cdot a^{\frac{4}{6}} = a^{\frac{9}{12}} \cdot a^{\frac{20}{12}} \cdot a^{\frac{8}{12}} = a^{\frac{27}{12}} = \sqrt[12]{a^{27}} = a^{\frac{3 \cdot 3}{12}} = a^{\frac{3}{4}}$

b) $\sqrt[3]{3a^2b} \cdot \sqrt{2ab^3} = (3a^2b)^{\frac{1}{3}} \cdot (2ab^3)^{\frac{1}{2}} = (3a^2b)^{\frac{2}{6}} \cdot (2ab^3)^{\frac{3}{6}} = \sqrt[6]{3^2 a^4 b^2 \cdot 2^3 a^3 b^9} = \sqrt[6]{2^3 3^2 a^7 b^{11}}$

c) $\sqrt[5]{2a^3b^4} : \sqrt[3]{4ab^2} = (2a^3b^4)^{\frac{1}{5}} : (4ab^2)^{\frac{1}{3}} = (2a^3b^4)^{\frac{3}{15}} : (4ab^2)^{\frac{5}{15}} = \sqrt[15]{\frac{2^3 a^9 b^{12}}{4^5 a^5 b^{10}}} = \sqrt[15]{\frac{2^3 a^9 b^{12}}{2^{10} a^5 b^{10}}} = \sqrt[15]{\frac{a^4 b^2}{2^7}}$

d) $\sqrt[3]{\sqrt{ab}} \cdot \sqrt{a\sqrt[3]{b}} = \left((ab)^{\frac{1}{2}}\right)^{\frac{1}{3}} \cdot \left(a(b)^{\frac{1}{3}}\right)^{\frac{1}{2}} = a^{\frac{1}{6}} b^{\frac{1}{6}} a^{\frac{1}{2}} b^{\frac{1}{6}} = a^{\frac{2}{3}} b^{\frac{1}{3}} = \sqrt[3]{a^2 b}$

Ejercicio 16:

Efectúa y simplifica.

a) $(2 + \sqrt{3})^2 - (2 + \sqrt{3}) \cdot (2 - \sqrt{3})$

b) $(3 + \sqrt{5}) \cdot (3 - \sqrt{5}) + (2 - 4\sqrt{5}) \cdot (2 + 4\sqrt{5})$

c) $(\sqrt{3} + \sqrt{5} - 4\sqrt{7}) \cdot (\sqrt{3} - \sqrt{5} + 4\sqrt{7})$

a) $(2 + \sqrt{3})^2 - (2 + \sqrt{3}) \cdot (2 - \sqrt{3}) = 4 + 4\sqrt{3} + 3 - 4 + 3 = 6 + 4\sqrt{3}$

b) $(3 + \sqrt{5}) \cdot (3 - \sqrt{5}) + (2 - 4\sqrt{5}) \cdot (2 + 4\sqrt{5}) = 9 - 5 + 4 - 80 = -72$

c) $(\sqrt{3} + \sqrt{5} - 4\sqrt{7}) \cdot (\sqrt{3} - \sqrt{5} + 4\sqrt{7}) = 3 - \sqrt{15} + 4\sqrt{21} + \sqrt{15} - 5 + 4\sqrt{35} - 4\sqrt{21} + 4\sqrt{35} - 112 = 109 + 8\sqrt{35}$

Ejercicio 17:

Efectúa y simplifica.

a) $\frac{\sqrt[4]{2^3} \cdot 2^{-4} \cdot \sqrt[3]{2}}{2^2 \cdot \sqrt{2} \cdot 2^{-\frac{5}{2}}}$

c) $\left(\sqrt{14 + \sqrt{7 - \sqrt[4]{81}}}\right)^{-\frac{1}{2}}$

b) $\left(81^{\frac{1}{4}} \cdot \sqrt[4]{\frac{1}{3}} \cdot \frac{1}{\sqrt[8]{3}}\right) : \sqrt{3}$

d) $\left(\sqrt{\frac{a}{9} + \frac{a}{16}}\right)^{-2}$

a) $\frac{\sqrt[4]{2^3} \cdot 2^{-4} \cdot \sqrt[3]{2}}{2^2 \cdot \sqrt{2} \cdot 2^{-\frac{5}{2}}} = \frac{2^{\frac{3}{4}} \cdot 2^{-4} \cdot 2^{\frac{1}{3}}}{2^2 \cdot 2^{\frac{1}{2}} \cdot 2^{-\frac{5}{2}}} = \frac{2^{\frac{13}{12}}}{2^4} = \frac{2^{\frac{13}{12}}}{2^{\frac{48}{12}}} = \sqrt[12]{2^{35}}$

b) $\left(81^{\frac{1}{4}} \cdot \sqrt[4]{\frac{1}{3}} \cdot \frac{1}{\sqrt[8]{3}}\right) : \sqrt{3} = \left(3 \cdot 3^{-\frac{1}{4}} \cdot 3^{-\frac{1}{8}}\right) : 3^{\frac{1}{2}} = 3^{\frac{5}{8}} : 3^{\frac{1}{2}} = 3^{\frac{1}{8}} = \sqrt[8]{3}$

c) $\left(\sqrt{14 + \sqrt{7 - \sqrt[4]{81}}}\right)^{-\frac{1}{2}} = \left(\sqrt{14 + \sqrt{7 - 3}}\right)^{-\frac{1}{2}} = \left(\sqrt{14 + 2}\right)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

d) $\left(\sqrt{\frac{a}{9} + \frac{a}{16}}\right)^{-2} = \left(\sqrt{\frac{16a + 9a}{144}}\right)^{-2} = \left(\sqrt{\frac{25a}{144}}\right)^{-2} = \left(\frac{5}{12}\sqrt{a}\right)^{-2} = \frac{144}{25a}$

Ejercicio 18:

Expresa el resultado como potencia.

a) $(\sqrt[3]{5} \cdot \sqrt{5})^6$ c) $\sqrt{\sqrt[3]{2^2}} \cdot \sqrt{\sqrt{\sqrt{2}}}$

b) $\sqrt[5]{3} \cdot \sqrt[5]{3^2} \sqrt{3}$ d) $\sqrt[3]{8\sqrt[5]{81}}$

a) $(\sqrt[3]{5} \cdot \sqrt{5})^6 = \left(5^{\frac{1}{3}} \cdot 5^{\frac{1}{2}}\right)^6 = \left(5^{\frac{5}{6}}\right)^6 = 5^5$

b) $\sqrt[5]{3} \cdot \sqrt[5]{3^2} \sqrt{3} = 3^{\frac{1}{5}} \cdot \left(3^2 \cdot 3^{\frac{1}{2}}\right)^{\frac{1}{5}} = 3^{\frac{1}{5}} \cdot 3^{\frac{2}{5}} \cdot 3^{\frac{1}{10}} = 3^{\frac{7}{10}}$

c) $\sqrt{\sqrt[3]{2^2}} \cdot \sqrt{\sqrt{\sqrt{2}}} = \left(2^{\frac{2}{3}}\right)^{\frac{1}{2}} \cdot \left(\left(2^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = 2^{\frac{1}{3}} \cdot 2^{\frac{1}{8}} = 2^{\frac{11}{24}}$

d) $\sqrt[3]{8\sqrt[5]{81}} = \left(2^3 (3^4)^{\frac{1}{5}}\right)^{\frac{1}{3}} = 2 \cdot 3^{\frac{4}{15}}$

Ejercicio 19:

Elimina las raíces del denominador.

a) $\frac{1}{\sqrt{2} + 1}$ c) $\frac{-5}{\sqrt{3} - 2}$ e) $\frac{7}{\sqrt{11} - 3}$

b) $\frac{3}{\sqrt{2} + \sqrt{3}}$ d) $\frac{4\sqrt{2}}{3\sqrt{2} - \sqrt{5}}$ f) $\frac{-5}{\sqrt{6} + \sqrt{7}}$

a) $\frac{1}{\sqrt{2} + 1} = \frac{\sqrt{2} - 1}{(\sqrt{2} + 1)(\sqrt{2} - 1)} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$

b) $\frac{3}{\sqrt{2} + \sqrt{3}} = \frac{3(\sqrt{2} - \sqrt{3})}{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})} = \frac{3(\sqrt{2} - \sqrt{3})}{2 - 3} = -3(\sqrt{2} - \sqrt{3})$

c) $\frac{-5}{\sqrt{3} - 2} = \frac{-5(\sqrt{3} + 2)}{(\sqrt{3} - 2)(\sqrt{3} + 2)} = \frac{-5\sqrt{3} - 10}{3 - 4} = 5\sqrt{3} + 10$

d) $\frac{4\sqrt{2}}{3\sqrt{2} - \sqrt{5}} = \frac{4\sqrt{2}(3\sqrt{2} + \sqrt{5})}{(3\sqrt{2} - \sqrt{5})(3\sqrt{2} + \sqrt{5})} = \frac{24 + 4\sqrt{10}}{18 - 5} = \frac{24 + 4\sqrt{10}}{13}$

e) $\frac{7}{\sqrt{11} - 3} = \frac{7(\sqrt{11} + 3)}{(\sqrt{11} - 3)(\sqrt{11} + 3)} = \frac{7\sqrt{11} + 21}{11 - 9} = \frac{7\sqrt{11} + 21}{2}$

f) $\frac{-5}{\sqrt{6} + \sqrt{7}} = \frac{-5(\sqrt{6} - \sqrt{7})}{(\sqrt{6} + \sqrt{7})(\sqrt{6} - \sqrt{7})} = \frac{-5\sqrt{6} + 5\sqrt{7}}{6 - 7} = 5\sqrt{6} - 5\sqrt{7}$