

**EJERCICIOS RESUELTOS DE POLINOMIOS**  
**REPASO 1º BACHILLERATO**

1) Dados los polinomios:

$$P(x) = 4x^2 - 1$$

$$Q(x) = x^3 - 3x^2 + 6x - 2$$

$$R(x) = 6x^2 + x + 1$$

$$S(x) = 1/2x^2 + 4$$

$$T(x) = 3/2x^2 + 5$$

$$U(x) = x^2 + 2$$

Calcular:

**a.-**  $P(x) + Q(x) =$

$$(4x^2 - 1) + (x^3 - 3x^2 + 6x - 2) = x^3 - 3x^2 + 4x^2 + 6x - 2 - 1 = \mathbf{x^3 + x^2 + 6x - 3}$$

**b.-**  $P(x) - U(x) =$

$$= (4x^2 - 1) - (x^2 + 2) = 4x^2 - 1 - x^2 - 2 = \mathbf{3x^2 - 3}$$

**c.-**  $P(x) + R(x) =$

$$= (4x^2 - 1) + (6x^2 + x + 1) = 4x^2 + 6x^2 + x - 1 + 1 = \mathbf{10x^2 + x}$$

**d.-**  $2P(x) - R(x) =$

$$= 2 \cdot (4x^2 - 1) - (6x^2 + x + 1) = 8x^2 - 2 - 6x^2 - x - 1 = \mathbf{2x^2 - x - 3}$$

**e.-**  $S(x) + T(x) + U(x) =$

$$= (1/2 x^2 + 4) + (3/2 x^2 + 5) + (x^2 + 2) = 1/2 x^2 + 3/2 x^2 + x^2 + 4 + 5 + 2 = \mathbf{3x^2 + 11}$$

**f.-**  $S(x) - T(x) + U(x) =$

$$= (1/2 x^2 + 4) - (3/2 x^2 + 5) + (x^2 + 2) = 1/2 x^2 + 4 - 3/2 x^2 - 5 + x^2 + 2 = \mathbf{1}$$

2) Multiplicar:

**a.-**  $(x^4 - 2x^2 + 2) \cdot (x^2 - 2x + 3) =$

$$= x^6 - 2x^5 + 3x^4 - 2x^4 + 4x^3 - 6x^2 + 2x^2 - 4x + 6 = x^6 - 2x^5 - 2x^4 + 3x^4 + 4x^3 + 2x^2 - 6x^2 - 4x + 6 = \mathbf{x^6 - 2x^5 + x^4 + 4x^3 - 4x^2 - 4x + 6}$$

**b.-**  $(3x^2 - 5x) \cdot (2x^3 + 4x^2 - x + 2) =$

$$= 6x^5 + 12x^4 - 3x^3 + 6x^2 - 10x^4 - 20x^3 + 5x^2 - 10x = 6x^5 + 12x^4 - 10x^4 - 3x^3 - 20x^3 + 6x^2 + 5x^2 - 10x = \mathbf{6x^5 + 2x^4 - 23x^3 + 11x^2 - 10x}$$

**c.-**  $(2x^2 - 5x + 6) \cdot (3x^4 - 5x^3 - 6x^2 + 4x - 3) =$

$$= 6x^6 - 10x^5 - 12x^4 + 8x^3 - 6x^2 - 15x^5 + 25x^4 + 30x^3 - 20x^2 + 15x + 18x^4 - 30x^3 - 36x^2 + 24x - 18 = \mathbf{6x^6 - 10x^5 - 15x^5 - 12x^4 + 25x^4 + 18x^4 + 8x^3 - 30x^3 + 30x^3 - 6x^2 - 20x^2 - 36x^2 + 15x + 24x - 18 = 6x^6 - 25x^5 + 31x^4 + 8x^3 - 62x^2 + 39x - 18}$$

3) Dividir:

**a.-**  $(x^4 - 2x^3 - 11x^2 + 30x - 20) : (x^2 + 3x - 2)$

$$\begin{array}{r}
 x^4 - 2x^3 - 11x^2 + 30x - 20 \\
 \underline{-x^4 - 3x^3 + 2x^2} \\
 -5x^3 - 9x^2 + 30x \\
 \underline{5x^3 + 15x^2 - 10x} \\
 6x^2 + 20x - 20 \\
 \underline{-6x^2 - 18x + 12} \\
 2x - 8
 \end{array}
 \quad
 \begin{array}{r}
 \underline{x^2 + 3x - 2} \\
 x^2 - 5x + 6
 \end{array}$$

b.-  $(x^6 + 5x^4 + 3x^2 - 2x) : (x^2 - x + 3)$

$$\begin{array}{r}
 x^6 \quad + 5x^4 \quad + 3x^2 - 2x \\
 \underline{-x^6 + x^5 - 3x^4} \\
 x^5 + 2x^4 \\
 \underline{-x^5 + x^4 - 3x^3} \\
 3x^4 - 3x^3 + 3x^2 \\
 \underline{-3x^4 + 3x^3 - 9x^2} \\
 -6x^2 - 2x \\
 \underline{6x^2 - 6x + 18}
 \end{array}
 \quad
 \begin{array}{r}
 \underline{x^2 - x + 3} \\
 x^4 + x^3 + 3x^2 - 6
 \end{array}$$

c.-  $P(x) = x^5 + 2x^3 - x - 8$        $Q(x) = x^2 - 2x + 1$

$$\begin{array}{r}
 x^5 \quad + 2x^3 \quad - x - 8 \\
 \underline{-x^5 + 2x^4 - x^3} \\
 2x^4 + x^3 \\
 \underline{-2x^4 + 4x^3 - 2x^2} \\
 5x^3 - 2x^2 - x \\
 \underline{-5x^3 + 10x^2 - 5x} \\
 8x^2 - 6x - 8 \\
 \underline{-8x^2 + 16x - 8} \\
 10x - 16
 \end{array}
 \quad
 \begin{array}{r}
 \underline{x^2 - 2x + 1} \\
 x^3 + 2x^2 + 5x + 8
 \end{array}$$

4) Divide por Ruffini:

a.-  $(x^3 + 2x + 70) : (x + 4)$

$$\begin{array}{r}
 1 \quad 0 \quad 2 \quad 70 \\
 -4 \quad -4 \quad 16 \quad -72 \\
 \hline
 1 \quad -4 \quad 18 \quad \underline{-2}
 \end{array}$$

$C(x) = x^2 - 4x + 18$        $R(x) = -2$

b.-  $(x^5 - 32) : (x - 2)$

$$\begin{array}{r}
 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad -32 \\
 2 \quad \quad 2 \quad 4 \quad 8 \quad 16 \quad 32 \\
 \hline
 1 \quad 2 \quad 4 \quad 8 \quad 16 \quad \underline{0}
 \end{array}$$

$C(x) = x^4 + 2x^3 + 4x^2 + 8x + 16$        $R = 0$



8) Descomponer factorialmente los siguientes polinomios:

a)  $12x^3 - 3x$

b)  $2x^4 + 12x^3 + 18x^2$

c)  $45x^2 - 120x + 80$

d)  $12x^3 + 12x^2 + 3x$

a)  $12x^3 - 3x = 3x(4x^2 - 1) = 3x(2x - 1)(2x + 1)$

b)  $2x^4 + 12x^3 + 18x^2 = 2x^2(x^2 + 6x + 9) = 2x^2(x + 3)^2$

c)  $45x^2 - 120x + 80 = 5(9x^2 - 24x + 16) = 5(3x - 4)^2$

d)  $12x^3 + 12x^2 + 3x = 3x(4x^2 + 4x + 1) = 3x(2x + 1)^2$

9) Descomponer en factores los siguientes polinomios:

a)  $x^3 - x^2 + 4x - 4$

b)  $x^3 - x - 6$

c)  $3x^4 + 15x^2$

d)  $x^4 - 16$

a)  $x^3 - x^2 + 4x - 4$

$$\begin{array}{r|rrrr} & 1 & -1 & 4 & -4 \\ 1 & & 1 & 0 & 4 \\ \hline & 1 & 0 & 4 & \boxed{0} \end{array} \rightarrow x^2 + 4 \text{ no tiene raíces reales.}$$

$x^3 - x^2 + 4x - 4 = (x - 1)(x^2 + 4)$

b)  $x^3 - x - 6$

$$\begin{array}{r|rrrr} & 1 & 0 & -1 & -6 \\ 2 & & 2 & 4 & 6 \\ \hline & 1 & 2 & 3 & \boxed{0} \end{array}$$

El polinomio  $x^2 + 2x + 3$  no tiene raíces reales, luego:

$x^3 - x - 6 = (x - 2)(x^2 + 2x + 3)$

c)  $3x^4 + 15x^2 = 3x^2(x^2 + 5)$

d)  $x^4 - 16 = (x^2 - 4) \cdot (x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$

10) Descomponer en factores:

a)  $x^2 - x - 6$

$x^2 - x - 6 = 0 \rightarrow$

$$x = \frac{1 \pm \sqrt{1^2 + 4 \cdot 6}}{2} = \frac{1 \pm \sqrt{1 + 24}}{2} = \frac{1 \pm 5}{2} = \begin{matrix} \nearrow x_1 = \frac{6}{2} = 3 \\ \searrow x_2 = \frac{-4}{2} = -2 \end{matrix}$$

$x^2 - x - 6 = (x + 2) \cdot (x - 3)$

b)  $x^3 + 3x^2 - 4x - 12$

Posibles ceros:  $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12\}$

$$\begin{array}{r|rrrr} & 1 & 3 & -4 & -12 \\ 2 & & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$(x - 2) \cdot (x^2 + 5x + 6) \rightarrow x^2 + 5x + 6 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 6}}{2} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2} = \frac{-5 \pm 1}{2} = \begin{matrix} \nearrow x_1 = \frac{-4}{2} = -2 \\ \searrow x_2 = \frac{-6}{2} = -3 \end{matrix}$$

$(x - 2) \cdot (x + 2) \cdot (x + 3)$

Las raíces son :  $x = 2, x = -2, x = -3$ .