EJERCICIOS RESUELTOS TRIGONOMETRÍA I

Cuestión 1:

a) Pasa a radianes los siguientes ángulos: 210° y 70°

b) Pasa a grados los ángulos: $\frac{7\pi}{6}$ rad y 3,5 rad

Solución:

$$a)\ 210^\circ = 210 \cdot \frac{\pi}{180}\ \text{rad} = \frac{7\pi}{6}\ \text{rad}$$

$$70^\circ = 70 \cdot \frac{\pi}{180}\ \text{rad} = \frac{7\pi}{18}\ \text{rad}$$

$$b)\ \frac{7\pi}{6}\ \text{rad} = \frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = 210^\circ$$

$$3{,}5\ \text{rad} = 3{,}5 \cdot \frac{180^\circ}{\pi} = 200^\circ\, 32'\, 7''$$

Cuestión 2:

Completa la siguiente tabla:

GRADOS	35º		120º	
RADIANES		2π / 3		2

Solución:

$$35^\circ = \frac{35 \cdot \pi}{180}\ \text{rad} = \frac{7\pi}{36}\ \text{rad}$$

$$\frac{2\pi}{3}\ \text{rad} = \frac{2\pi}{3} \cdot \frac{180^\circ}{\pi} = 120^\circ \quad \rightarrow \quad 120^\circ = \frac{2\pi}{3}\ \text{rad}$$

$$2\ \text{rad} = 2 \cdot \frac{180^\circ}{\pi} = 114^\circ\, 35'\, 30''$$

Por tanto:

GRADOS	**35º**	120º	**120º**	114º35'30"
RADIANES	7π / 36	**2π / 3**	2π / 3	**2**

Cuestión 3:

a) Expresa en grados los siguientes ángulos dados en radianes $\frac{5\pi}{6}$ y 3

b) Expresa en radianes los ángulos: 225° y 100°

Solución:

a) $\dfrac{5\pi}{6}\ \text{rad} = \dfrac{5\pi}{6}\cdot\dfrac{180^\circ}{\pi} = 150^\circ$

$3\ \text{rad} = 3\cdot\dfrac{180^\circ}{\pi} = 171^\circ 53'14''$

b) $225^\circ = 225\cdot\dfrac{\pi}{180}\ \text{rad} = \dfrac{5\pi}{4}\ \text{rad}$

$100^\circ = 100\cdot\dfrac{\pi}{180}\ \text{rad} = \dfrac{5\pi}{9}\ \text{rad}$

<u>Cuestión 4</u>:

Calcular todas las razones trigonométricas en los siguientes casos:

a. $\text{sen}\ \alpha = \dfrac{1}{3} : \alpha < 90^\circ$

b. $\cos\alpha = -\dfrac{3}{5} : \dfrac{\pi}{2} < \alpha < \pi$

c. $\text{tag}\ \alpha = 2 : 180^\circ < \alpha < 270^\circ$

d. $\text{cosec}\ \alpha = -\dfrac{3}{2} : 270^\circ < \alpha < 360^\circ$

e. $\sec\alpha = -2 :\ \pi < \alpha < \dfrac{3\pi}{2}$

f. $\text{cotag}\ \alpha = -1 : 270^\circ < \alpha < 360^\circ$

Solución.

a. $\text{sen}\ \alpha = \dfrac{1}{3}$ Sí $\alpha < 90^\circ \Rightarrow \alpha \in 1^\circ$ Cuadrante: $\begin{cases} \text{sen}\ \alpha\ ;\ \text{cosec}\ \alpha > 0 \\ \cos\alpha\ ;\ \sec\alpha > 0 \\ \text{tag}\ \alpha\ ;\ \text{cotag}\ \alpha > 0 \end{cases}$

Conocido el valor del seno se calcula el coseno mediante la ecuación fundamental.

$$\text{sen}^2\alpha + \cos^2\alpha = 1$$

$$\cos\alpha = \pm\sqrt{1-\text{sen}^2\alpha} = +\sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

Conocido el seno y el coseno se calcula la tangente por su definición.

$$\text{tag}\ \alpha = \frac{\text{sen}\ \alpha}{\cos\alpha} = \frac{1/3}{2\sqrt{2}/3} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

Conocidas las razones directas (seno, coseno y tangente) se calculan la inversas (cosecante, secante y cotangente) mediante su definición.

$$\text{cosec}\ \alpha = \frac{1}{\text{sen}\ \alpha} = \frac{1}{1/3} = 3 \qquad \sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{2\sqrt{2}/3} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\text{cotag}\ \alpha = \frac{1}{\text{tag}\ \alpha} = \frac{1}{\sqrt{2}/4} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

b. $\cos\alpha = -\frac{3}{5}$: Sí $\frac{\pi}{2} < \alpha < \pi \Rightarrow \alpha \in 2^{o}$ Cuadrante: $\begin{cases} \text{sen}\,\alpha;\ \text{cosec}\,\alpha > 0 \\ \cos\alpha;\ \sec\alpha < 0 \\ \text{tag}\,\alpha;\ \text{cotag}\,\alpha < 0 \end{cases}$

Conocido el valor del coseno se calcula el seno mediante la ecuación fundamental.

$$\text{sen}^2\alpha + \cos^2\alpha = 1$$

$$\text{sen}\,\alpha = \pm\sqrt{1-\cos^2\alpha} = +\sqrt{1-\left(-\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Conocido el seno y el coseno se calcula la tangente por su definición.

$$\text{tag}\,\alpha = \frac{\text{sen}\,\alpha}{\cos\alpha} = \frac{4/5}{-3/5} = -\frac{4}{3}$$

Conocidas las razones directas (seno, coseno y tangente) se calculan la inversas (cosecante, secante y cotangente) mediante su definición.

$$\text{cosec}\,\alpha = \frac{1}{\text{sen}\,\alpha} = \frac{1}{4/5} = \frac{5}{4} \qquad \sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{-3/5} = -\frac{5}{3}$$

$$\text{cotag}\,\alpha = \frac{1}{\text{tag}\,\alpha} = \frac{1}{-4/3} = -\frac{3}{4}$$

c. $\text{tag}\,\alpha = 2$: Sí $180^{o} < \alpha < 270^{o} \Rightarrow \alpha \in 3^{o}$ Cuadrante: $\begin{cases} \text{sen}\,\alpha;\ \text{cosec}\,\alpha < 0 \\ \cos\alpha;\ \sec\alpha < 0 \\ \text{tag}\,\alpha;\ \text{cotag}\,\alpha > 0 \end{cases}$

Conocido el valor de la tangente se obtienen la cotangente y la secante.

$$\text{cotag}\,\alpha = \frac{1}{\text{tag}\,\alpha} = \frac{1}{2}$$

$$\text{tag}^2\alpha + 1 = \sec^2\alpha : \ \sec\alpha = \pm\sqrt{\text{tag}^2\alpha + 1} = -\sqrt{2^2+1} = -\sqrt{5}$$

Con la secante se obtiene el coseno

$$\sec\alpha = \frac{1}{\cos\alpha} : \ \cos\alpha = \frac{1}{\sec\alpha} = \frac{1}{-\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

Conocidas la tangente y el coseno se obtiene el seno mediante la definición de tangente.

$$\text{tag}\,\alpha = \frac{\text{sen}\,\alpha}{\cos\alpha} : \ \text{sen}\,\alpha = \cos\alpha \cdot \text{tag}\,\alpha = -\frac{\sqrt{5}}{5}\cdot 2 = -\frac{2\sqrt{5}}{5}$$

Por último del seno se obtiene la cosecante.

$$\text{cosec}\,\alpha = \frac{1}{\text{sen}\,\alpha} = \frac{1}{-2\sqrt{5}/5} = -\frac{5}{2\sqrt{5}} = -\frac{\sqrt{5}}{2}$$

d. $\operatorname{cosec}\alpha = -\dfrac{3}{2}$: Si $270^\circ < \alpha < 360^\circ \Rightarrow \alpha \in 4^\circ$ Cuadrante: $\begin{cases} \operatorname{sen}\alpha\,;\ \operatorname{cosec}\alpha < 0 \\ \cos\alpha\,;\ \sec\alpha > 0 \\ \operatorname{tag}\alpha\,;\ \operatorname{cotag}\alpha < 0 \end{cases}$

De la definición de cosecante se obtienen el seno y la cotangente.

$$\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha} : \ \operatorname{sen}\alpha = \frac{1}{\operatorname{cosec}\alpha} = \frac{1}{-3/2} = -\frac{2}{3}$$

Conocido el valor del seno se calcula el coseno mediante la ecuación fundamental.

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

$$\cos\alpha = \pm\sqrt{1-\operatorname{sen}^2\alpha} = +\sqrt{1-\left(-\frac{2}{3}\right)^2} = \sqrt{1-\frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

Conocido el seno y el coseno se calcula la tangente por su definición.

$$\operatorname{tag}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{-2/3}{\sqrt{5}/3} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

Conocidas las razones directas (coseno y tangente) se calculan la inversas (secante y cotangente) mediante su definición.

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{\sqrt{5}/3} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5} \qquad \operatorname{cotag}\alpha = \frac{1}{\operatorname{tag}\alpha} = \frac{1}{-2\sqrt{5}/5} = -\frac{5}{2\sqrt{5}} = -\frac{\sqrt{5}}{2}$$

e. $\sec\alpha = -2$: Si $\pi < \alpha < \dfrac{3\pi}{2} \Rightarrow \alpha \in 3^\circ$ Cuadrante: $\begin{cases} \operatorname{sen}\alpha\,;\ \operatorname{cosec}\alpha < 0 \\ \cos\alpha\,;\ \sec\alpha < 0 \\ \operatorname{tag}\alpha\,;\ \operatorname{cotag}\alpha > 0 \end{cases}$

Conocida la secante se calcula el coseno y la tangente.

$$\sec\alpha = \frac{1}{\cos\alpha} : \ \cos\alpha = \frac{1}{\sec\alpha} = \frac{1}{-2} = -\frac{1}{2}$$

$$\operatorname{tag}^2\alpha + 1 = \sec^2\alpha : \ \operatorname{tag}\alpha = \pm\sqrt{\sec^2\alpha - 1} = +\sqrt{(-2)^2-1} = \sqrt{3}$$

Conocidas la tangente y el coseno se obtiene el seno mediante la definición de tangente.

$$\operatorname{tag}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} : \ \operatorname{sen}\alpha = \cos\alpha \cdot \operatorname{tag}\alpha = -\frac{1}{2}\cdot\sqrt{3} = -\frac{\sqrt{3}}{2}$$

Conocidas las razones directas (seno y tangente) se calculan la inversas (cosecante y cotangente) mediante su definición.

$$\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} : \ \operatorname{cotag}\alpha = \frac{1}{\operatorname{tag}\alpha} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

f. $\text{cotag}\ \alpha = -1$: Si $270^\circ < \alpha < 360^\circ \Rightarrow \alpha \in 4^\circ$ Cuadrante: $\begin{cases} \text{sen}\ \alpha\ ;\ \text{cosec}\ \alpha < 0 \\ \cos\alpha\ ;\ \sec\alpha > 0 \\ \text{tag}\ \alpha\ ;\ \text{cotag}\ \alpha < 0 \end{cases}$

Conocida la cotangente se calcula la tangente y la cosecante.

$$\text{cotag}\ \alpha = \frac{1}{\text{tag}\ \alpha} \qquad \text{tag}\ \alpha = \frac{1}{\text{cotag}\ \alpha} = \frac{1}{-1} = -1$$

$$\text{cotag}^2\alpha + 1 = \text{cos ec}^2\alpha \qquad \text{cosec}\ \alpha = \pm\sqrt{\text{cotag}^2\alpha + 1} = -\sqrt{(-1)^2 + 1} = -\sqrt{2}$$

Conocida la cosecante se calcula el seno

$$\text{cosec}\ \alpha = \frac{1}{\text{sen}\ \alpha} \qquad \text{sen}\ \alpha = \frac{1}{\text{cosec}\ \alpha} = \frac{1}{-\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Con el seno y la tangente se calcula el coseno con la definición de tangente.

$$\text{tag}\ \alpha = \frac{\text{sen}\ \alpha}{\cos\alpha} \qquad \cos\alpha = \frac{\text{sen}\ \alpha}{\text{tag}\ \alpha} = \frac{-\sqrt{2}/2}{-1} = \frac{\sqrt{2}}{2}$$

Conocido el coseno se calcula la secante.

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Cuestión 5: Sabiendo que cos α = ¼ , y que 270º <α <360°. Calcular las restantes razones trigonométricas del ángulo α.

$$\text{sen}\ \alpha = -\sqrt{1-\left(\frac{1}{4}\right)^2} = -\frac{\sqrt{15}}{4} \qquad \text{cosec}\ \alpha = -\frac{4\sqrt{15}}{15}$$

$$\cos\alpha = \frac{1}{4} \qquad \sec\alpha = 4$$

$$\text{tg}\ \alpha = -\frac{\frac{\sqrt{15}}{4}}{\frac{1}{4}} = -\sqrt{15} \qquad \text{cotg}\ \alpha = -\frac{\sqrt{15}}{15}$$

Cuestión 6: Sabiendo que tg α = 2, y que 180º < α <270°. Calcular las restantes razones trigonométricas del ángulo α.

$$\sec\alpha = -\sqrt{1+4} = -\sqrt{5} \qquad \cos\alpha = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\text{sen}\ \alpha = 2\cdot\left(-\frac{\sqrt{5}}{5}\right) = -\frac{2\sqrt{5}}{5} \qquad \text{cosec}\ \alpha = -\frac{\sqrt{5}}{2}$$

$$\text{tg}\ \alpha = 2 \qquad \text{cotg}\ \alpha = \frac{1}{2}$$

Cuestión 7: Sabiendo que sec α = 2, 0< α < π/2, calcular las restantes razones trigonométricas.

$$\cos\ \alpha = \frac{1}{2} \qquad \sec\ \alpha = 2$$

$$\text{sen}\ \alpha = \sqrt{1-\left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} \qquad \text{cosec}\ \alpha = \frac{2\sqrt{3}}{3}$$

$$\text{tg}\ \alpha = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \qquad \text{cotg}\ \alpha = \frac{\sqrt{3}}{3}$$

Cuestión 8: Calcula las razones de los siguientes ángulos:

a) 225°

$$\sin(225°) = \sin(180°+45°) = -\sin 45° = -\frac{\sqrt{2}}{2}$$

$$\cos(225°) = \cos(180°+45°) = -\cos 45° = -\frac{\sqrt{2}}{2}$$

$$\tan(225°) = \tan(180°+45°) = \tan 45° = 1$$

b) 330°

$$\sin(330°) = \sin(360°-30°) = -\sin 30° = -\frac{1}{2}$$

$$\cos(330°) = \cos(360°-30°) = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan(330°) = \tan(360°-30°) = -\tan 30° = -\frac{\sqrt{3}}{3}$$

c) 655°

$$\begin{array}{l|l} 2655° & 360° \\ \hline 135° & 7 \end{array}$$

$$sen 2655° = sen 135° = sen(180° - 45°) = sen 45° = \frac{\sqrt{2}}{2}$$

$$\cos\ 2655° = \cos\ 135° = \cos\ (180° - 45°) = -\cos\ 45° = -\frac{\sqrt{2}}{2}$$

$$tg 2655° = -1$$

d) −840º

$$\begin{array}{ll} -840^\circ & \underline{|\,360^\circ} \\ -120^\circ & \quad -2 \end{array}$$

$$\sin(-840^\circ) = \sin(-120^\circ) = -\sin(180^\circ - 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos(-840^\circ) = \cos(-120^\circ) = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$$

$$\tan(-840^\circ) = \tan(-120^\circ) = -\tan(120^\circ) = \sqrt{3}$$

<u>Cuestión 9</u>:

Calcula las razones trigonométricas de 140° y de 220°, sabiendo que:

$$sen 40^\circ = 0{,}64;\ \cos 40^\circ = 0{,}77;\ tg\, 40^\circ = 0{,}84$$

Solución:

Como $140^\circ = 180^\circ - 40^\circ$ y $220^\circ = 180^\circ + 40^\circ$, entonces

$$sen 140^\circ = sen 40^\circ = 0{,}64$$
$$\cos 140^\circ = -\cos 40^\circ = -0{,}77$$
$$tg 140^\circ = -tg 40^\circ = -0{,}84$$

$$sen 220^\circ = -sen 40^\circ = -0{,}64$$
$$\cos 220^\circ = -\cos 40^\circ = -0{,}77$$
$$tg 220^\circ = tg 40^\circ = 0{,}84$$

<u>Cuestión 10</u>:

Sabiendo que *sen* 50º = 0,77, *cos* 50º = 0,64 y *tg* 50º = 1,19, calcula (sin utilizar las teclas trigonométricas de la calculadora):

a) $\cos 130^\circ$ b) $tg\, 310^\circ$ c) $\cos 230^\circ$ d) $sen 310^\circ$

Solución:

a) $\cos 130^\circ = \cos(180^\circ - 150^\circ) = -\cos 50^\circ = -0{,}64$

b) $tg 310^\circ = tg(360^\circ - 50^\circ) = -tg 50^\circ = -1{,}19$

c) $\cos 230^\circ = \cos(180^\circ + 50^\circ) = -\cos 50^\circ = -0{,}64$

d) $sen 310^\circ = sen(360^\circ - 50^\circ) = -sen 50^\circ = -0{,}77$

Cuestión 11:

Si $sen\ \alpha = 0{,}35$ y $0° < \alpha < 90°$ halla (sin calcular α):

a) $sen(180° - \alpha)$ b) $cos(180° + \alpha)$

Solución:

$$a)\ sen(180° - \alpha) = sen\,\alpha = 0{,}35$$
$$b)\ cos(180° + \alpha) = -\cos\alpha$$

Necesitamos saber cuánto vale $cos\ \alpha$:

$$sen^2\alpha + cos^2\alpha = 1 \quad \to \quad 0{,}35^2 + cos^2\alpha = 1$$
$$0{,}1225 + cos^2\alpha = 1 \quad \to \quad cos^2\alpha = 0{,}8775$$
$$cos\alpha = 0{,}94 \ \text{(es positivo, pues } 0° < \alpha < 90°\text{)}$$

$$\text{Por tanto } cos(180° + \alpha) = -cos\alpha = -0{,}94$$

Cuestión 12:

Si $tg\,\alpha = \frac{1}{3}$ y α es un ángulo que está en el primer cuadrante, calcula (sin hallar α):

a) $tg(180° - \alpha)$ b) $tg(180° + \alpha)$ c) $tg(360° - \alpha)$ d) $tg(360° + \alpha)$

Solución:

$$a)\ tg(180° - \alpha) = -tg\,\alpha = -\frac{1}{3}$$
$$b)\ tg(180° + \alpha) = tg\,\alpha = \frac{1}{3}$$
$$c)\ tg(360° - \alpha) = -tg\,\alpha = -\frac{1}{3}$$
$$d)\ tg(360° + \alpha) = tg\,\alpha = \frac{1}{3}$$

Cuestión 13: Comprobar las identidades:

a) $tg\,\alpha + \text{cotg}\ \alpha = \sec\alpha \cdot \text{cosec}\,\alpha$

$$tg\,\alpha + \text{cotg}\ \alpha = \frac{sen\,\alpha}{\cos\alpha} + \frac{\cos\alpha}{sen\,\alpha} = \frac{sen^2\alpha + \cos^2\alpha}{\cos\alpha \cdot sen\,\alpha} =$$

$$= \frac{1}{\cos\alpha \cdot sen\,\alpha} = \sec\alpha \cdot \text{cosec}\,\alpha$$

b) $\text{cotg}^2\,a = \cos^2 a + (\text{cotg}\ a \cdot \cos a)^2$

$$\cos^2 a + (\text{cotg}\ a \cdot \cos a)^2 = \cos^2 a + \text{cotg}^2\ a \cdot \cos^2 a =$$

$$\cos^2 a\left(1+\text{cotg}^2\ a\right)=\cos^2 a\cdot\text{cosec}^2 a=\frac{\cos^2 a}{\text{sen}^2 a}=\text{cotg}^2 a$$

c) $\dfrac{1}{\sec^2 a}=\text{sen}^2 a\cdot\cos^2 a+\cos^4 a$

$$\text{sen}^2 a\cdot\cos^2 a+\cos^4 a=\cos^2 a\left(\text{sen}^2 a+\cos^2 a\right)=\cos^2 a=\frac{1}{\sec^2 a}$$

d) $\text{cotg}\, a\cdot\sec a=\text{cosec}\, a$

$$\text{cotg}\, a\cdot\sec a=\frac{\cos a}{\text{sen}\, a}\cdot\frac{1}{\cos a}=\frac{1}{\text{sen}\, a}=\text{cosec}\, a$$

e) $\sec^2 a+\text{cosec}^2 a=\dfrac{1}{\text{sen}^2 a\cdot\cos^2 a}$

$$\sec^2 a+\text{cosec}^2 a=\frac{1}{\cos^2 a}+\frac{1}{\text{sen}^2 a}=\frac{\text{sen}^2 a+\cos^2 a}{\text{sen}^2 a\cdot\cos^2 a}=\frac{1}{\text{sen}^2 a\cdot\cos^2 a}$$

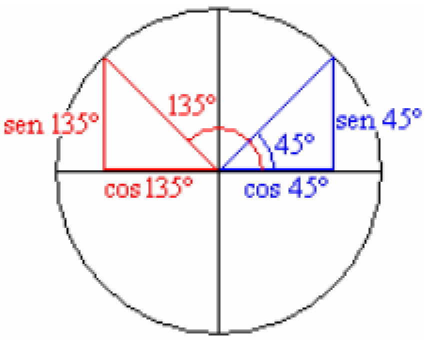
Cuestión 14:

Calcular las razones trigonométricas de los siguientes ángulos en función de sus ángulos asociados agudos.

- **a)** 135º
- **b)** 120º
- **c)** 330º
- **d)** 240º
- **e)** 150º
- **f)** 1290º
- **g)** Sabiendo que tg 18º = 0,32 calcular la razones trigonométricas de los siguientes ángulos:
  - **i)** 72º
  - **ii)** 108º
  - **iii)** 162º
  - **iv)** 198º
  - **v)** 252º
  - **vi)** 288º
  - **vii)** 342º

Solución.

a. 135º es suplementario con 45º (135º + 45º =180º). Las razones trigonométricas de 135º están relacionadas con las de 45º, la forma más sencilla de encontrar esta relación es de forma gráfica.

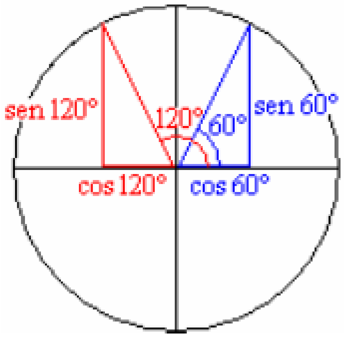


$$\text{sen}\,135º=\text{sen}\,45=\frac{\sqrt{2}}{2}$$

$$\cos 135º=-\cos 45º=-\frac{\sqrt{2}}{2}$$

$$\text{tg}\,135º=\frac{\text{sen}\,135º}{\cos 135º}=\frac{\text{sen}\,45º}{-\cos 45º}=-\text{tg}\,45º=-1$$

b. 120º es suplementario con 60º (120º + 60º =180º). Las razones trigonométricas de 120º están relacionadas con las de 60º, la forma más sencilla de encontrar esta relación es de forma gráfica.

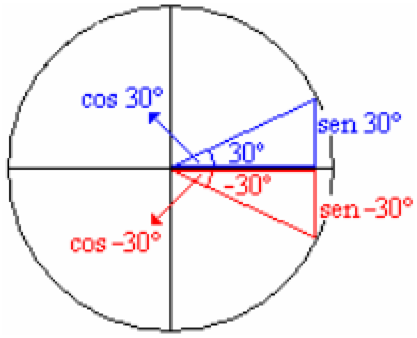


$$\text{sen}\,120° = \text{sen}\,60° = \frac{\sqrt{3}}{2}$$

$$\cos 120° = -\cos 60° = -\frac{1}{2}$$

$$\text{tg}\,135° = \frac{\text{sen}\,120°}{\cos 120°} = \frac{\text{sen}\,60°}{-\cos 60°} = -\text{tg}\,60° = -\sqrt{3}$$

c. 330º equivalente a −30º, asociado a 30º

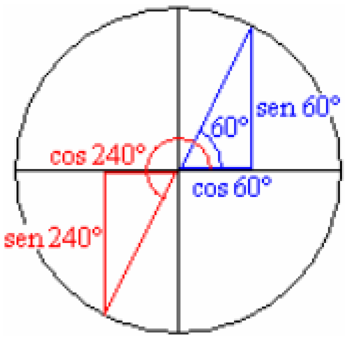


$$\text{sen} - 30° = -\text{sen}\,30° = -\frac{1}{2}$$

$$\cos - 30° = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\text{tg}\,(-30°) = \frac{\text{sen}\,(-30°)}{\cos\,(-30°)} = -\text{tg}30° = -\frac{\sqrt{3}}{3}$$

d. 240º se asocia a 60º porque se diferencia del él en 180º (240º = 60º +180º).

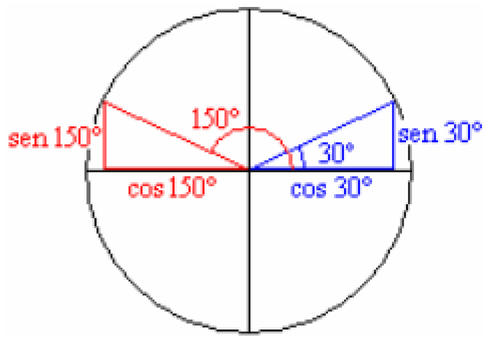


$$\text{sen}\,240° = -\text{sen}\,60 = -\frac{\sqrt{3}}{2}$$

$$\cos 240° = -\cos 60° = -\frac{1}{2}$$

$$\text{tg}\,240° = \frac{\text{sen}\,240°}{\cos 240°} = \frac{-\text{sen}\,60°}{-\cos 60°} = \text{tg}\,60 = \sqrt{3}$$

e. 150º suplementario de 30º (150º + 30º = 180º)



$$\text{sen}\,150° = \text{sen}\,30° = \frac{1}{2}$$

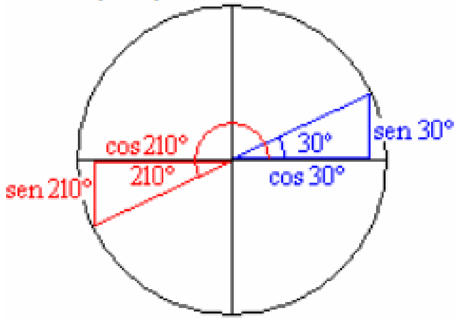
$$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$$

$$\text{tg}\,150° = \frac{\text{sen}\,150°}{\cos 150°} = \frac{\text{sen}\,30°}{-\cos 30°} = -\text{tg}\,30° = -\frac{\sqrt{3}}{3}$$

f. 1290. Por ser un ángulo superior a 360º, se divide por 360 y nos quedamos con el resto.

$$1260º = 3 \times 360º + 210$$

Las razones trigonométricas de 1290º coinciden con las de 210º, (relación entre las razones trigonométricas de ángulos que se diferencian en un número entero de vueltas, 360º ó 2π radianes) y las de este (210º) se relacionan con las de 30º (210º = 180º+30º).



$$\text{sen}\,1290º = \text{sen}\,(360º \times 3 + 210º) = \text{sen}\,210º = -\text{sen}\,30º = -\frac{1}{2}$$

$$\cos 1290º = \cos(360º \times 3 + 210º) = \cos 210º = -\cos 30º = -\frac{\sqrt{3}}{2}$$

$$\text{tg}\,1290º = \text{tg}\,(360º \times 3 + 210º) = \text{tg}\,210º = \frac{\text{sen}\,210º}{\cos 210} = \frac{-\text{sen}\,30º}{-\cos 30º} = \text{tg}\,30º = \frac{\sqrt{3}}{2}$$

g. Lo primero es calcular el seno y el coseno de 18º conocida la tangente (tg 18º = 0,32). Por ser un ángulo del primer cuadrante, todas sus razones trigonométricas son positivas.

Conocido el valor de la tangente se obtienen la secante.

$$\text{tag}^2 18º + 1 = \sec^2 18º : \ \sec 18º = \pm\sqrt{\text{tag}^2 18º + 1} = +\sqrt{0{,}32^2 + 1} = 1{,}05$$

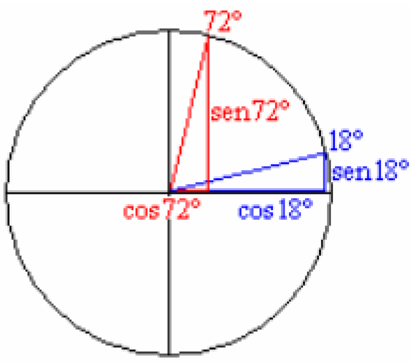
Con la secante se obtiene el coseno

$$\sec 18º = \frac{1}{\cos 18º} : \ \cos 18º = \frac{1}{\sec 18º} = \frac{1}{1{,}05} = 0{,}95$$

Conocidas la tangente y el coseno se obtiene el seno mediante la definición de tangente.

$$\text{tag}\,18º = \frac{\text{sen}\,18º}{\cos 18º} : \ \text{sen}\,18º = \cos 18 \cdot \text{tag}\,18 = 0{,}95 \cdot 0{,}32 = 0{,}30$$

i. 72º = 90º − 18º

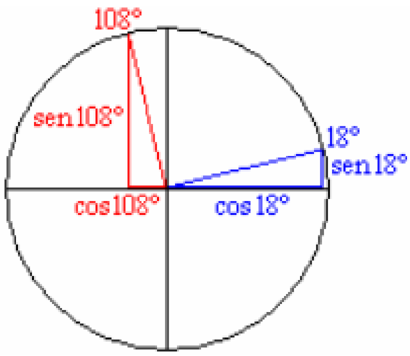


$$\text{sen}\,72º = \cos 18º = 0{,}95$$

$$\cos 72º = \text{sen}\,18º = 0{,}30$$

$$\text{tg}\,72º = \frac{\text{sen}\,72º}{\cos 72º} = \frac{\cos 18º}{\text{sen}\,18º} = \frac{1}{\text{tg}\,18º} = \frac{1}{0{,}32} = 3{,}12$$

ii. 108º = 90º + 18º

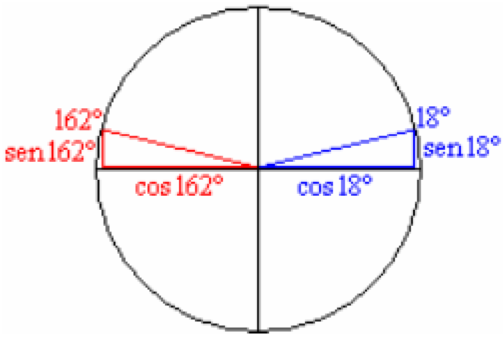


$$\text{sen}108º = \cos 18 = 0{,}95$$

$$\cos 108º = -\text{sen}18º = -0{,}30$$

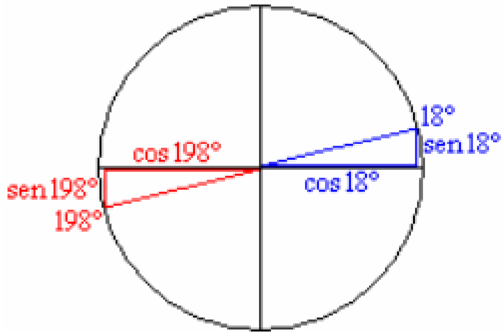
$$\text{tg}\,108º = \frac{\text{sen}108º}{\cos 108º} = \frac{\cos 18º}{-\text{sen}\,18º} = -\frac{1}{\text{tg}\,18º} = -\frac{1}{0{,}32} = -3{,}12$$

iii. $162° = 180° - 18°$



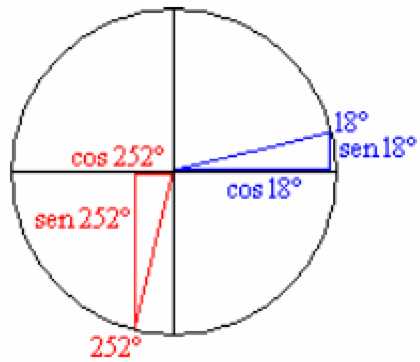
$$\text{sen}\,162° = \text{sen}\,18° = 0{,}30$$
$$\cos 162° = -\cos 18° = -0{,}95$$
$$\text{tg}\,162° = \frac{\text{sen}162°}{\cos 162°} = \frac{\text{sen}\,18°}{-\cos 18°} = -\text{tg}\,18° = -0{,}32$$

iv. $198° = 180° + 18°$



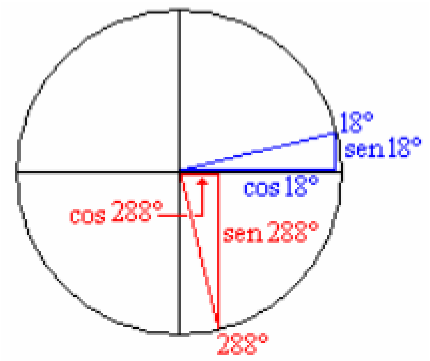
$$\text{sen}\,198° = -\text{sen}\,18° = -0{,}30$$
$$\cos 198° = -\cos 18° = -0{,}95$$
$$\text{tg}\,198° = \frac{\text{sen}198°}{\cos 198°} = \frac{-\text{sen}\,18°}{-\cos 18°} = \text{tg}\,18° = 0{,}32$$

v. $252° = 270° - 18°$



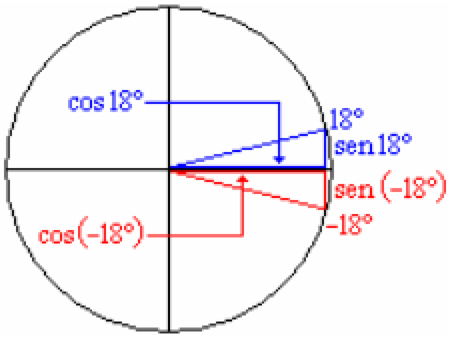
$$\text{sen}\,252° = -\cos 18 = -0{,}95$$
$$\cos 252° = -\text{sen}18° = -0{,}30$$
$$\text{tg}\,252° = \frac{\text{sen}\,252°}{\cos 252°} = \frac{-\cos 18°}{-\text{sen}\,18°} = \frac{1}{\text{tg}\,18°} = \frac{1}{0{,}32} = 3{,}12$$

vi. $298° = 270° + 18°$



$$\text{sen}\,288° = -\cos 18 = -0{,}95$$
$$\cos 288° = \text{sen}18° = 0{,}30$$
$$\text{tg}\,288° = \frac{\text{sen}\,288°}{\cos 288°} = \frac{-\cos 18°}{\text{sen}\,18°} = -\frac{1}{\text{tg}\,18°} = -\frac{1}{0{,}32} = -3{,}12$$

vii. $342^\circ = -18^\circ$



$$\text{sen}\,(-18^\circ) = -\text{sen}\,18^\circ = -0{,}30$$
$$\cos(-18^\circ) = \cos 18^\circ = 0{,}95$$
$$\text{tg}\,(-18^\circ) = \frac{\text{sen}(-18^\circ)}{\cos(-18)^\circ} = \frac{-\text{sen}\,18^\circ}{\cos 18^\circ} = -\text{tg}\,18^\circ = -0{,}32$$

C**<u>uestión 15</u>**: Simplificar las siguientes expresiones:

a)

$$\text{sen}\,\alpha \cdot \frac{1}{\text{tg}\,\alpha}$$

$$\text{sen}\,\alpha \cdot \frac{1}{\text{tg}\,\alpha} = \text{sen}\,\alpha \cdot \frac{1}{\text{sen}\alpha / \cos\alpha} = \text{sen}\,\alpha \cdot \frac{\cos\alpha}{\text{sen}\,\alpha} = \cos\alpha$$

b)

$$\sqrt{1-\text{sen}\alpha} \cdot \sqrt{1+\text{sen}\alpha}$$

$$\sqrt{1-\text{sen}\alpha} \cdot \sqrt{1+\text{sen}\alpha} = \sqrt{(1-\text{sen}\alpha)\cdot(1+\text{sen}\alpha)} = \sqrt{1^2-\text{sen}^2\alpha} = \sqrt{\cos^2\alpha} = \cos\alpha$$

c)

$$\frac{\cos^2\alpha - \text{sen}^2\alpha}{\cos^4\alpha - \text{sen}^4\alpha}$$

$$\frac{\cos^2\alpha - \text{sen}^2\alpha}{\cos^4\alpha - \text{sen}^4\alpha} = \frac{\cos^2\alpha - \text{sen}^2\alpha}{(\cos^2\alpha - \text{sen}^2\alpha)\cdot(\cos^2\alpha + \text{sen}^2\alpha)} = \frac{1}{\cos^2\alpha + \text{sen}^2\alpha} = \frac{1}{1} = 1$$

d)

$$\frac{\cos^2\alpha - \text{sen}^2\alpha}{\text{sen}^2\alpha - \cos^2\alpha}$$

$$\frac{\cos^2\alpha - \text{sen}^2\alpha}{\text{sen}^2\alpha - \cos^2\alpha} = \frac{-(\text{sen}^2\alpha - \cos^2\alpha)}{\text{sen}^2\alpha - \cos^2\alpha} = -1$$

e)

$$\frac{\sec^2\alpha + \cos^2\alpha}{\sec^2\alpha - \cos^2\alpha}$$

$$\frac{\sec^2\alpha + \cos^2\alpha}{\sec^2\alpha - \cos^2\alpha} = \frac{\frac{1}{\cos^2\alpha} + \cos^2\alpha}{\frac{1}{\cos^2\alpha} - \cos^2\alpha} = \frac{\frac{1+\cos^4\alpha}{\cos^2\alpha}}{\frac{1-\cos^4\alpha}{\cos^2\alpha}} = \frac{1+\cos^4\alpha}{1-\cos^4\alpha}$$

f)

$$\frac{\text{cosec}\alpha}{1+\text{ctg}^2\alpha}$$

$$\frac{\operatorname{cosec}\alpha}{1+\operatorname{ctg}^2\alpha}=\frac{\frac{1}{\operatorname{sen}\alpha}}{1+\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}}=\frac{\frac{1}{\operatorname{sen}\alpha}}{\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\operatorname{sen}^2\alpha}}=\frac{\frac{1}{\operatorname{sen}\alpha}}{\frac{1}{\operatorname{sen}^2\alpha}}=\frac{\operatorname{sen}^2\alpha}{\operatorname{sen}\alpha}=\operatorname{sen}\alpha$$

g)

$$\frac{\cos^2\alpha}{1-\operatorname{sen}\alpha}$$

$$\frac{\cos^2\alpha}{1-\operatorname{sen}\alpha}=\frac{1-\operatorname{sen}^2\alpha}{1-\operatorname{sen}\alpha}=\frac{(1+\operatorname{sen}\alpha)\cdot(1-\operatorname{sen}\alpha)}{1-\operatorname{sen}\alpha}=1+\operatorname{sen}\alpha$$

Cuestión 16:

Demostrar si son verdaderas o falsas las siguientes ecuaciones:

a) $\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{ctg}\alpha+\operatorname{ctg}\beta}=\operatorname{tg}\alpha\cdot\operatorname{tg}\beta$

$$\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{ctg}\alpha+\operatorname{ctg}\beta}=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\frac{1}{\operatorname{tg}\alpha}+\frac{1}{\operatorname{tg}\beta}}=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\frac{\operatorname{tg}\beta+\operatorname{tg}\alpha}{\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}}=\operatorname{tg}\alpha\cdot\operatorname{tg}\beta$$

b) $\frac{\operatorname{ctg}\alpha+\operatorname{tg}\alpha}{\operatorname{ctg}\alpha-\operatorname{tg}\alpha}=\frac{1}{\cos^2\alpha-\operatorname{sen}^2\alpha}$

$$\frac{\operatorname{ctg}\alpha+\operatorname{tg}\alpha}{\operatorname{ctg}\alpha-\operatorname{tg}\alpha}=\frac{\frac{\cos\alpha}{\operatorname{sen}\alpha}+\frac{\operatorname{sen}\alpha}{\cos\alpha}}{\frac{\cos\alpha}{\operatorname{sen}\alpha}-\frac{\operatorname{sen}\alpha}{\cos\alpha}}=\frac{\frac{\cos\alpha\cdot\cos\alpha+\operatorname{sen}\alpha\cdot\operatorname{sen}\alpha}{\operatorname{sen}\alpha\cdot\cos\alpha}}{\frac{\cos\alpha\cdot\cos\alpha-\operatorname{sen}\alpha\cdot\operatorname{sen}\alpha}{\operatorname{sen}\alpha\cdot\cos\alpha}}=\frac{\cos^2\alpha+\operatorname{sen}^2\alpha}{\cos^2\alpha-\operatorname{sen}^2\alpha}=\frac{1}{\cos^2\alpha-\operatorname{sen}^2\alpha}=$$

$$=\frac{1}{\cos 2\alpha}=\sec 2\alpha$$

c) $\operatorname{tg}\alpha+\operatorname{ctg}\alpha=\sec\alpha\cdot\operatorname{cosec}\alpha$

$$\operatorname{tg}\alpha+\operatorname{ctg}\alpha=\frac{\operatorname{sen}\alpha}{\cos\alpha}+\frac{\cos\alpha}{\operatorname{sen}\alpha}=\frac{\operatorname{sen}\alpha\cdot\operatorname{sen}\alpha+\cos\alpha\cdot\cos\alpha}{\cos\alpha\cdot\operatorname{sen}\alpha}=\frac{1}{\cos\alpha\cdot\operatorname{sen}\alpha}=\frac{1}{\cos\alpha}\cdot\frac{1}{\operatorname{sen}\alpha}=\sec\alpha\cdot\operatorname{cosec}\alpha$$

d) $\operatorname{ctg}^2\alpha-\cos^2\alpha=\operatorname{ctg}^2\alpha\cdot\cos^2\alpha$

$$\operatorname{ctg}^2\alpha-\cos^2\alpha=\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}-\cos^2\alpha=\frac{\cos^2\alpha-\cos^2\alpha\cdot\operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha}=\frac{\cos^2\alpha\cdot(1-\operatorname{sen}^2\alpha)}{\operatorname{sen}^2\alpha}=$$

$$=\frac{\cos^2\alpha\cdot\cos^2\alpha}{\operatorname{sen}^2\alpha}=\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}\cdot\cos^2\alpha=\operatorname{ctg}^2\alpha\cdot\cos^2\alpha$$

e) $\operatorname{sen}^2\alpha-\cos^2\beta=\operatorname{sen}^2\beta-\cos^2\alpha$

$$\operatorname{sen}^2\alpha-\cos^2\beta=(1-\cos^2\alpha)-(1-\operatorname{sen}^2\beta)=1-\cos^2\alpha-1+\operatorname{sen}^2\beta=\operatorname{sen}^2\beta-\cos^2\alpha$$

f) $\frac{\operatorname{sen}\alpha\cdot\cos\alpha}{\cos^2\alpha-\operatorname{sen}^2\alpha}=\frac{\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$

$$\frac{\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}=\frac{\frac{\operatorname{sen}\alpha}{\cos\alpha}}{1-\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}}=\frac{\frac{\operatorname{sen}\alpha}{\cos\alpha}}{\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{\cos^2\alpha}}=\frac{\operatorname{sen}\alpha\cdot\cos^2\alpha}{\cos\alpha\cdot(\cos^2\alpha-\operatorname{sen}^2\alpha)}=\frac{\operatorname{sen}\alpha\cdot\cos\alpha}{\cos^2\alpha-\operatorname{sen}^2\alpha}$$

g) $\dfrac{1+\text{tg}\alpha}{1-\text{tg}\alpha}=\dfrac{\cos\alpha+\text{sen}\alpha}{\cos\alpha-\text{sen}\alpha}$

$$\frac{1+\text{tg}\,\alpha}{1-\text{tg}\,\alpha}=\frac{1+\dfrac{\text{sen}\,\alpha}{\cos\alpha}}{1-\dfrac{\text{sen}\,\alpha}{\cos\alpha}}=\frac{\dfrac{\cos\alpha+\text{sen}\,\alpha}{\cos\alpha}}{\dfrac{\cos\alpha-\text{sen}\,\alpha}{\cos\alpha}}=\frac{\cos\alpha+\text{sen}\,\alpha}{\cos\alpha-\text{sen}\,\alpha}$$

h) $\cos^2\alpha\cdot\cos^2\beta-\text{sen}^2\alpha\cdot\text{sen}^2\beta=\cos^2\alpha-\text{sen}^2\beta$

$$\cos^2\alpha\cdot\cos^2\beta-\text{sen}^2\alpha\cdot\text{sen}^2\beta=\cos^2\alpha\cdot\left(1-\text{sen}^2\beta\right)-\left(1-\cos^2\alpha\right)\cdot\text{sen}^2\beta=$$
$$=\cos^2\alpha-\cos^2\alpha\cdot\text{sen}^2\beta-\text{sen}^2\beta+\cos^2\alpha\cdot\text{sen}^2\beta=\cos^2\alpha-\text{sen}^2\beta$$

i) $\text{sen}\,\alpha\cdot\cos\alpha\cdot\text{tg}\,\alpha\cdot\text{ctg}\,\alpha\cdot\sec\alpha\cdot\text{cosec}\,\alpha=1$

$$\text{sen}\,\alpha\cdot\cos\alpha\cdot\text{tg}\,\alpha\cdot\text{ctg}\,\alpha\cdot\sec\alpha\cdot\text{cosec}\,\alpha=\text{sen}\,\alpha\cdot\cos\alpha\cdot\frac{\text{sen}\,\alpha}{\cos\alpha}\cdot\frac{\cos\alpha}{\text{sen}\,\alpha}\cdot\frac{1}{\cos\alpha}\cdot\frac{1}{\text{sen}\,\alpha}=1$$

j) $(\text{sen}\,\alpha+\cos\alpha)^2+(\text{sen}\,\alpha-\cos\alpha)^2=2$

$$(\text{sen}\,\alpha+\cos\alpha)^2+(\text{sen}\,\alpha-\cos\alpha)^2=\text{sen}^2\alpha+2\text{sen}\alpha\cdot\cos\alpha+\cos^2\alpha+\text{sen}^2\alpha-2\text{sen}\alpha\cdot\cos\alpha+\cos^2\alpha=$$
$$=\left(\text{sen}^2\alpha+\cos^2\alpha\right)+\left(\text{sen}^2\alpha+\cos^2\alpha\right)=1+1=2$$

k) $\text{ctg}\alpha-\dfrac{\text{ctg}^2\alpha-1}{\text{ctg}\alpha}=\text{tg}\alpha$

$$\text{ctg}\alpha-\frac{\text{ctg}^2\alpha-1}{\text{ctg}\alpha}=\frac{1}{\text{tg}\,\alpha}-\frac{\dfrac{1}{\text{tg}^2\alpha}-1}{\dfrac{1}{\text{tg}\,\alpha}}=\frac{1}{\text{tg}\,\alpha}-\frac{\dfrac{1-\text{tg}^2\alpha}{\text{tg}^2\alpha}}{\dfrac{1}{\text{tg}\,\alpha}}=\frac{1}{\text{tg}\,\alpha}-\frac{\text{tg}\,\alpha\cdot\left(1-\text{tg}^2\alpha\right)}{\text{tg}^2\alpha}=$$
$$=\frac{1}{\text{tg}\,\alpha}-\frac{1-\text{tg}^2\alpha}{\text{tg}\,\alpha}=\frac{1-\left(1-\text{tg}^2\alpha\right)}{\text{tg}\,\alpha}=\frac{1-1+\text{tg}^2\alpha}{\text{tg}\,\alpha}=\frac{\text{tg}^2\alpha}{\text{tg}\,\alpha}=\text{tg}\,\alpha$$

l) $\dfrac{1+\text{tg}^2\alpha}{\text{ctg}\alpha}=\dfrac{\text{tg}\alpha}{\cos^2\alpha}$

$$\frac{1+\text{tg}^2\alpha}{\text{ctg}\alpha}=\frac{1+\dfrac{\text{sen}^2\alpha}{\cos^2\alpha}}{\dfrac{1}{\text{tg}\,\alpha}}=\frac{\dfrac{\cos^2\alpha+\text{sen}^2\alpha}{\cos^2\alpha}}{\dfrac{1}{\text{tg}\,\alpha}}=\frac{\dfrac{1}{\cos^2\alpha}}{\dfrac{1}{\text{tg}\,\alpha}}=\frac{\text{tg}\alpha}{\cos^2\alpha}$$

m) $\dfrac{\text{sen}\,\alpha+\text{ctg}\,\alpha}{\text{tg}\,\alpha+\text{cosec}\,\alpha}=\cos\alpha$

$$\frac{\text{sen}\,\alpha+\text{ctg}\,\alpha}{\text{tg}\,\alpha+\text{cosec}\,\alpha}=\frac{\text{sen}\,\alpha+\dfrac{\cos\alpha}{\text{sen}\,\alpha}}{\dfrac{\text{sen}\,\alpha}{\cos\alpha}+\dfrac{1}{\text{sen}\,\alpha}}=\frac{\dfrac{\text{sen}^2\alpha+\cos\alpha}{\text{sen}\,\alpha}}{\dfrac{\text{sen}^2\alpha+\cos\alpha}{\cos\alpha\cdot\text{sen}\,\alpha}}=\frac{\dfrac{1}{\text{sen}\,\alpha}}{\dfrac{1}{\cos\alpha\cdot\text{sen}\,\alpha}}=\frac{\cos\alpha\cdot\text{sen}\,\alpha}{\text{sen}\,\alpha}=\cos\alpha$$

n) $\dfrac{1-\text{sen}\,\alpha}{\cos\alpha}=\dfrac{\cos\alpha}{1+\text{sen}\,\alpha}$

$$\frac{1-\text{sen}\,\alpha}{\cos\alpha}=\frac{(1-\text{sen}\,\alpha)\cdot(1+\text{sen}\,\alpha)}{\cos\alpha\cdot(1+\text{sen}\,\alpha)}=\frac{1^2-\text{sen}^2\alpha}{\cos\alpha\cdot(1+\text{sen}\,\alpha)}=\frac{\cos^2\alpha}{\cos\alpha\cdot(1+\text{sen}\,\alpha)}=\frac{\cos\alpha}{1+\text{sen}\,\alpha}$$