

EJERCICIOS RESUELTOS**UNIDAD 6: TRIGONOMETRÍA I****Cuestión 1:**a) Pasa a radianes los siguientes ángulos: 210° y 70° b) Pasa a grados los ángulos: $\frac{7\pi}{6}$ rad y $3,5$ rad**Solución:**

a) $210^\circ = 210 \cdot \frac{\pi}{180} \text{ rad} = \frac{7\pi}{6} \text{ rad}$

$70^\circ = 70 \cdot \frac{\pi}{180} \text{ rad} = \frac{7\pi}{18} \text{ rad}$

b) $\frac{7\pi}{6} \text{ rad} = \frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = 210^\circ$

$3,5 \text{ rad} = 3,5 \cdot \frac{180^\circ}{\pi} = 200^\circ 32' 7''$

Cuestión 2:

Completa la siguiente tabla:

| | | | | |
|----------|------------|------------|-------------|---|
| GRADOS | 35° | | 120° | |
| RADIANES | | $2\pi / 3$ | | 2 |

Solución:

$35^\circ = \frac{35 \cdot \pi}{180} \text{ rad} = \frac{7\pi}{36} \text{ rad}$

$\frac{2\pi}{3} \text{ rad} = \frac{2\pi}{3} \cdot \frac{180^\circ}{\pi} = 120^\circ \rightarrow 120^\circ = \frac{2\pi}{3} \text{ rad}$

$2 \text{ rad} = 2 \cdot \frac{180^\circ}{\pi} = 114^\circ 35' 30''$

Por tanto:

| | | | | |
|----------|-------------|-------------|-------------|----------------------|
| GRADOS | 35° | 120° | 120° | $114^\circ 35' 30''$ |
| RADIANES | $7\pi / 36$ | $2\pi / 3$ | $2\pi / 3$ | 2 |

Cuestión 3:

- a) Expresa en grados los siguientes ángulos dados en radianes $\frac{5\pi}{6}$ y 3
- b) Expresa en radianes los ángulos: 225° y 100°

Solución:

$$a) \frac{5\pi}{6} \text{ rad} = \frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} = 150^\circ$$

$$3 \text{ rad} = 3 \cdot \frac{180^\circ}{\pi} = 171^\circ 53' 14''$$

$$b) 225^\circ = 225 \cdot \frac{\pi}{180} \text{ rad} = \frac{5\pi}{4} \text{ rad}$$

$$100^\circ = 100 \cdot \frac{\pi}{180} \text{ rad} = \frac{5\pi}{9} \text{ rad}$$

Cuestión 4:

Calcular todas las razones trigonométricas en los siguientes casos:

a. $\text{sen } \alpha = \frac{1}{3} : \alpha < 90^\circ$

b. $\text{cos } \alpha = -\frac{3}{5} : \frac{\pi}{2} < \alpha < \pi$

c. $\text{tag } \alpha = 2 : 180^\circ < \alpha < 270^\circ$

d. $\text{cosec } \alpha = -\frac{3}{2} : 270^\circ < \alpha < 360^\circ$

e. $\text{sec } \alpha = -2 : \pi < \alpha < \frac{3\pi}{2}$

f. $\text{cotag } \alpha = -1 : 270^\circ < \alpha < 360^\circ$

Solución.

$$a. \quad \text{sen } \alpha = \frac{1}{3} \quad \text{Si } \alpha < 90^\circ \Rightarrow \alpha \in 1^\circ \text{ Cuadrante: } \begin{cases} \text{sen } \alpha ; \text{ cosec } \alpha > 0 \\ \text{cos } \alpha ; \text{ sec } \alpha > 0 \\ \text{tag } \alpha ; \text{ cotag } \alpha > 0 \end{cases}$$

Conocido el valor del seno se calcula el coseno mediante la ecuación fundamental.

$$\text{sen}^2 \alpha + \text{cos}^2 \alpha = 1$$

$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha} = + \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

Conocido el seno y el coseno se calcula la tangente por su definición.

$$\operatorname{tag} \alpha = \frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha} = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

Conocidas las razones directas (seno, coseno y tangente) se calculan la inversas (cosecante, secante y cotangente) mediante su definición.

$$\operatorname{cosec} \alpha = \frac{1}{\operatorname{sen} \alpha} = \frac{1}{\frac{1}{3}} = 3 \qquad \operatorname{sec} \alpha = \frac{1}{\operatorname{cos} \alpha} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\operatorname{cotag} \alpha = \frac{1}{\operatorname{tag} \alpha} = \frac{1}{\frac{\sqrt{2}}{4}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

b. $\cos \alpha = -\frac{3}{5}$: Si $\frac{\pi}{2} < \alpha < \pi \Rightarrow \alpha \in 2^\circ \text{ Cuadrante}$: $\begin{cases} \operatorname{sen} \alpha ; \operatorname{cosec} \alpha > 0 \\ \operatorname{cos} \alpha ; \operatorname{sec} \alpha < 0 \\ \operatorname{tag} \alpha ; \operatorname{cotag} \alpha < 0 \end{cases}$

Conocido el valor del coseno se calcula el seno mediante la ecuación fundamental.

$$\begin{aligned} \operatorname{sen}^2 \alpha + \operatorname{cos}^2 \alpha &= 1 \\ \operatorname{sen} \alpha &= \pm \sqrt{1 - \operatorname{cos}^2 \alpha} = + \sqrt{1 - \left(-\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5} \end{aligned}$$

Conocido el seno y el coseno se calcula la tangente por su definición.

$$\operatorname{tag} \alpha = \frac{\operatorname{sen} \alpha}{\operatorname{cos} \alpha} = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$$

Conocidas las razones directas (seno, coseno y tangente) se calculan la inversas (cosecante, secante y cotangente) mediante su definición.

$$\operatorname{cosec} \alpha = \frac{1}{\operatorname{sen} \alpha} = \frac{1}{\frac{4}{5}} = \frac{5}{4} \qquad \operatorname{sec} \alpha = \frac{1}{\operatorname{cos} \alpha} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$$

$$\operatorname{cotag} \alpha = \frac{1}{\operatorname{tag} \alpha} = \frac{1}{-\frac{4}{3}} = -\frac{3}{4}$$

$$c. \quad \text{tag } \alpha = 2: \text{ Si } 180^\circ < \alpha < 270^\circ \Rightarrow \alpha \in 3^\circ \text{ Cuadrante: } \begin{cases} \text{sen } \alpha; \text{ cosec } \alpha < 0 \\ \text{cos } \alpha; \text{ sec } \alpha < 0 \\ \text{tag } \alpha; \text{ cotag } \alpha > 0 \end{cases}$$

Conocido el valor de la tangente se obtienen la cotangente y la secante.

$$\text{cotag } \alpha = \frac{1}{\text{tag } \alpha} = \frac{1}{2}$$

$$\text{tag}^2 \alpha + 1 = \text{sec}^2 \alpha : \text{sec } \alpha = \pm \sqrt{\text{tag}^2 \alpha + 1} = -\sqrt{2^2 + 1} = -\sqrt{5}$$

Con la secante se obtiene el coseno

$$\text{sec } \alpha = \frac{1}{\text{cos } \alpha} : \text{cos } \alpha = \frac{1}{\text{sec } \alpha} = \frac{1}{-\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

Conocidas la tangente y el coseno se obtiene el seno mediante la definición de tangente.

$$\text{tag } \alpha = \frac{\text{sen } \alpha}{\text{cos } \alpha} : \text{sen } \alpha = \text{cos } \alpha \cdot \text{tag } \alpha = -\frac{\sqrt{5}}{5} \cdot 2 = -\frac{2\sqrt{5}}{5}$$

Por último del seno se obtiene la cosecante.

$$\text{cosec } \alpha = \frac{1}{\text{sen } \alpha} = \frac{1}{-\frac{2\sqrt{5}}{5}} = -\frac{5}{2\sqrt{5}} = -\frac{\sqrt{5}}{2}$$

$$d. \quad \text{cosec } \alpha = -\frac{3}{2} : \text{ Si } 270^\circ < \alpha < 360^\circ \Rightarrow \alpha \in 4^\circ \text{ Cuadrante: } \begin{cases} \text{sen } \alpha; \text{ cosec } \alpha < 0 \\ \text{cos } \alpha; \text{ sec } \alpha > 0 \\ \text{tag } \alpha; \text{ cotag } \alpha < 0 \end{cases}$$

De la definición de cosecante se obtienen el seno y la cotangente.

$$\text{cosec } \alpha = \frac{1}{\text{sen } \alpha} : \text{sen } \alpha = \frac{1}{\text{cosec } \alpha} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$$

Conocido el valor del seno se calcula el coseno mediante la ecuación fundamental.

$$\begin{aligned} \text{sen}^2 \alpha + \text{cos}^2 \alpha &= 1 \\ \text{cos } \alpha &= \pm \sqrt{1 - \text{sen}^2 \alpha} = +\sqrt{1 - \left(-\frac{2}{3}\right)^2} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3} \end{aligned}$$

Conocido el seno y el coseno se calcula la tangente por su definición.

$$\text{tag } \alpha = \frac{\text{sen } \alpha}{\text{cos } \alpha} = \frac{-\frac{2}{3}}{\frac{\sqrt{5}}{3}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

Conocidas las razones directas (coseno y tangente) se calculan la inversas (secante y cotangente) mediante su definición.

$$\text{sec } \alpha = \frac{1}{\text{cos } \alpha} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5} \quad \text{cotag } \alpha = \frac{1}{\text{tag } \alpha} = \frac{1}{-\frac{2\sqrt{5}}{5}} = -\frac{5}{2\sqrt{5}} = -\frac{\sqrt{5}}{2}$$

$$e. \quad \sec \alpha = -2 : \text{Si } \pi < \alpha < \frac{3\pi}{2} \Rightarrow \alpha \in 3^{\circ} \text{ Cuadrante: } \begin{cases} \text{sen } \alpha ; \text{ cosec } \alpha < 0 \\ \text{cos } \alpha ; \text{ sec } \alpha < 0 \\ \text{tag } \alpha ; \text{ cotag } \alpha > 0 \end{cases}$$

Conocida la secante se calcula el coseno y la tangente.

$$\sec \alpha = \frac{1}{\cos \alpha} : \cos \alpha = \frac{1}{\sec \alpha} = \frac{1}{-2} = -\frac{1}{2}$$

$$\text{tag}^2 \alpha + 1 = \sec^2 \alpha : \text{tag } \alpha = \pm \sqrt{\sec^2 \alpha - 1} = +\sqrt{(-2)^2 - 1} = \sqrt{3}$$

Conocidas la tangente y el coseno se obtiene el seno mediante la definición de tangente.

$$\text{tag } \alpha = \frac{\text{sen } \alpha}{\text{cos } \alpha} : \text{sen } \alpha = \text{cos } \alpha \cdot \text{tag } \alpha = -\frac{1}{2} \cdot \sqrt{3} = -\frac{\sqrt{3}}{2}$$

Conocidas las razones directas (seno y tangente) se calculan la inversas (cosecante y cotangente) mediante su definición.

$$\text{cosec } \alpha = \frac{1}{\text{sen } \alpha} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} : \text{cotag } \alpha = \frac{1}{\text{tag } \alpha} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$f. \quad \text{cotag } \alpha = -1 : \text{Si } 270^{\circ} < \alpha < 360^{\circ} \Rightarrow \alpha \in 4^{\circ} \text{ Cuadrante: } \begin{cases} \text{sen } \alpha ; \text{ cosec } \alpha < 0 \\ \text{cos } \alpha ; \text{ sec } \alpha > 0 \\ \text{tag } \alpha ; \text{ cotag } \alpha < 0 \end{cases}$$

Conocida la cotangente se calcula la tangente y la cosecante.

$$\text{cotag } \alpha = \frac{1}{\text{tag } \alpha} \quad \text{tag } \alpha = \frac{1}{\text{cotag } \alpha} = \frac{1}{-1} = -1$$

$$\text{cotag}^2 \alpha + 1 = \text{cosec}^2 \alpha \quad \text{cosec } \alpha = \pm \sqrt{\text{cotag}^2 \alpha + 1} = -\sqrt{(-1)^2 + 1} = -\sqrt{2}$$

Conocida la cosecante se calcula el seno

$$\text{cosec } \alpha = \frac{1}{\text{sen } \alpha} \quad \text{sen } \alpha = \frac{1}{\text{cosec } \alpha} = \frac{1}{-\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Con el seno y la tangente se calcula el coseno con la definición de tangente.

$$\text{tag } \alpha = \frac{\text{sen } \alpha}{\text{cos } \alpha} \quad \text{cos } \alpha = \frac{\text{sen } \alpha}{\text{tag } \alpha} = \frac{-\sqrt{2}/2}{-1} = \frac{\sqrt{2}}{2}$$

Conocido el coseno se calcula la secante.

$$\sec \alpha = \frac{1}{\text{cos } \alpha} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Cuestión 5: Sabiendo que $\cos \alpha = \frac{1}{4}$, y que $270^{\circ} < \alpha < 360^{\circ}$. Calcular las restantes razones trigonométricas del ángulo α .

$$\operatorname{sen} \alpha = -\sqrt{1 - \left(\frac{1}{4}\right)^2} = -\frac{\sqrt{15}}{4}$$

$$\operatorname{cosec} \alpha = -\frac{4\sqrt{15}}{15}$$

$$\operatorname{cos} \alpha = \frac{1}{4}$$

$$\operatorname{sec} \alpha = 4$$

$$\operatorname{tg} \alpha = -\frac{\frac{\sqrt{15}}{4}}{\frac{1}{4}} = -\sqrt{15}$$

$$\operatorname{cotg} \alpha = -\frac{\sqrt{15}}{15}$$

Cuestión 6: Sabiendo que $\operatorname{tg} \alpha = 2$, y que $180^\circ < \alpha < 270^\circ$. Calcular las restantes razones trigonométricas del ángulo α .

$$\operatorname{sec} \alpha = -\sqrt{1 + 4} = -\sqrt{5}$$

$$\operatorname{cos} \alpha = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\operatorname{sen} \alpha = 2 \cdot \left(-\frac{\sqrt{5}}{5}\right) = -\frac{2\sqrt{5}}{5}$$

$$\operatorname{cosec} \alpha = -\frac{\sqrt{5}}{2}$$

$$\operatorname{tg} \alpha = 2$$

$$\operatorname{cotg} \alpha = \frac{1}{2}$$

Cuestión 7: Sabiendo que $\operatorname{sec} \alpha = 2$, $0 < \alpha < \pi/2$, calcular las restantes razones trigonométricas.

$$\operatorname{cos} \alpha = \frac{1}{2}$$

$$\operatorname{sec} \alpha = 2$$

$$\operatorname{sen} \alpha = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$$

$$\operatorname{cosec} \alpha = \frac{2\sqrt{3}}{3}$$

$$\operatorname{tg} \alpha = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\operatorname{cotg} \alpha = \frac{\sqrt{3}}{3}$$

Cuestión 8: Calcula las razones de los siguientes ángulos:

a) 225°

$$\sin(225^\circ) = \sin(180^\circ + 45^\circ) = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\cos(225^\circ) = \cos(180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\tan(225^\circ) = \tan(180^\circ + 45^\circ) = \tan 45^\circ = 1$$

b) 330°

$$\sin(330^\circ) = \sin(360^\circ - 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$$

$$\cos(330^\circ) = \cos(360^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan(330^\circ) = \tan(360^\circ - 30^\circ) = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$$

c) 655°

$$\begin{array}{r} 2655^\circ \\ 135^\circ \end{array} \quad \begin{array}{r} | 360^\circ \\ 7 \end{array}$$

$$\operatorname{sen} 2655^\circ = \operatorname{sen} 135^\circ = \operatorname{sen}(180^\circ - 45^\circ) = \operatorname{sen} 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos 2655^\circ = \cos 135^\circ = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\operatorname{tg} 2655^\circ = -1$$

d) -840°

$$\begin{array}{r} -840^\circ \\ -120^\circ \end{array} \quad \begin{array}{r} | 360^\circ \\ -2 \end{array}$$

$$\sin(-840^\circ) = \sin(-120^\circ) = -\sin(180^\circ - 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos(-840^\circ) = \cos(-120^\circ) = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$$

$$\tan(-840^\circ) = \tan(-120^\circ) = -\tan(120^\circ) = \sqrt{3}$$

Cuestión 9:

Calcula las razones trigonométricas de 140° y de 220° , sabiendo que:

$$\operatorname{sen} 40^\circ = 0,64; \operatorname{cos} 40^\circ = 0,77; \operatorname{tg} 40^\circ = 0,84$$

Solución:

Como $140^\circ = 180^\circ - 40^\circ$ y $220^\circ = 180^\circ + 40^\circ$, entonces

$$\operatorname{sen}140^\circ = \operatorname{sen}40^\circ = 0,64$$

$$\operatorname{cos}140^\circ = -\operatorname{cos}40^\circ = -0,77$$

$$\operatorname{tg}140^\circ = -\operatorname{tg}40^\circ = -0,84$$

$$\operatorname{sen}220^\circ = -\operatorname{sen}40^\circ = -0,64$$

$$\operatorname{cos}220^\circ = -\operatorname{cos}40^\circ = -0,77$$

$$\operatorname{tg}220^\circ = \operatorname{tg}40^\circ = 0,84$$

Cuestión 10:

Sabiendo que $\operatorname{sen} 50^\circ = 0,77$, $\operatorname{cos} 50^\circ = 0,64$ y $\operatorname{tg} 50^\circ = 1,19$, calcula (sin utilizar las teclas trigonométricas de la calculadora):

- a) $\operatorname{cos}130^\circ$ b) $\operatorname{tg}310^\circ$ c) $\operatorname{cos}230^\circ$ d) $\operatorname{sen}310^\circ$

Solución:

$$\text{a) } \operatorname{cos}130^\circ = \operatorname{cos}(180^\circ - 50^\circ) = -\operatorname{cos}50^\circ = -0,64$$

$$\text{b) } \operatorname{tg}310^\circ = \operatorname{tg}(360^\circ - 50^\circ) = -\operatorname{tg}50^\circ = -1,19$$

$$\text{c) } \operatorname{cos}230^\circ = \operatorname{cos}(180^\circ + 50^\circ) = -\operatorname{cos}50^\circ = -0,64$$

$$\text{d) } \operatorname{sen}310^\circ = \operatorname{sen}(360^\circ - 50^\circ) = -\operatorname{sen}50^\circ = -0,77$$

Cuestión 11:

Si $\operatorname{sen} \alpha = 0,35$ y $0^\circ < \alpha < 90^\circ$ halla (sin calcular α):

- a) $\operatorname{sen}(180^\circ - \alpha)$ b) $\operatorname{cos}(180^\circ + \alpha)$

Solución:

$$\text{a) } \operatorname{sen}(180^\circ - \alpha) = \operatorname{sen} \alpha = 0,35$$

$$\text{b) } \operatorname{cos}(180^\circ + \alpha) = -\operatorname{cos} \alpha$$

Necesitamos saber cuánto vale $\operatorname{cos} \alpha$:

$$\operatorname{sen}^2 \alpha + \operatorname{cos}^2 \alpha = 1 \rightarrow 0,35^2 + \operatorname{cos}^2 \alpha = 1$$

$$0,1225 + \operatorname{cos}^2 \alpha = 1 \rightarrow \operatorname{cos}^2 \alpha = 0,8775$$

$$\operatorname{cos} \alpha = 0,94 \text{ (es positivo, pues } 0^\circ < \alpha < 90^\circ \text{)}$$

$$\text{Por tanto } \operatorname{cos}(180^\circ + \alpha) = -\operatorname{cos} \alpha = -0,94$$

Cuestión 12:

Si $\operatorname{tg} \alpha = \frac{1}{3}$ y α es un ángulo que está en el primer cuadrante, calcula (sin hallar α):

a) $\operatorname{tg}(180^\circ - \alpha)$ b) $\operatorname{tg}(180^\circ + \alpha)$ c) $\operatorname{tg}(360^\circ - \alpha)$ d) $\operatorname{tg}(360^\circ + \alpha)$

Solución:

$$\text{a) } \operatorname{tg}(180^\circ - \alpha) = -\operatorname{tg} \alpha = -\frac{1}{3}$$

$$\text{b) } \operatorname{tg}(180^\circ + \alpha) = \operatorname{tg} \alpha = \frac{1}{3}$$

$$\text{c) } \operatorname{tg}(360^\circ - \alpha) = -\operatorname{tg} \alpha = -\frac{1}{3}$$

$$\text{d) } \operatorname{tg}(360^\circ + \alpha) = \operatorname{tg} \alpha = \frac{1}{3}$$

Cuestión 13: Comprobar las identidades:

a) $\operatorname{tg} \alpha + \operatorname{cotg} \alpha = \sec \alpha \cdot \operatorname{cosec} \alpha$

$$\operatorname{tg} \alpha + \operatorname{cotg} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} + \frac{\cos \alpha}{\operatorname{sen} \alpha} = \frac{\operatorname{sen}^2 \alpha + \cos^2 \alpha}{\cos \alpha \cdot \operatorname{sen} \alpha} =$$

$$= \frac{1}{\cos \alpha \cdot \operatorname{sen} \alpha} = \sec \alpha \cdot \operatorname{cosec} \alpha$$

b) $\operatorname{cotg}^2 a = \cos^2 a + (\operatorname{cotg} a \cdot \cos a)^2$

$$\cos^2 a + (\operatorname{cotg} a \cdot \cos a)^2 = \cos^2 a + \operatorname{cotg}^2 a \cdot \cos^2 a =$$

$$\cos^2 a (1 + \operatorname{cotg}^2 a) = \cos^2 a \cdot \operatorname{cosec}^2 a = \frac{\cos^2 a}{\operatorname{sen}^2 a} = \operatorname{cotg}^2 a$$

c) $\frac{1}{\sec^2 a} = \operatorname{sen}^2 a \cdot \cos^2 a + \cos^4 a$

$$\operatorname{sen}^2 a \cdot \cos^2 a + \cos^4 a = \cos^2 a (\operatorname{sen}^2 a + \cos^2 a) = \cos^2 a = \frac{1}{\sec^2 a}$$

d) $\operatorname{cotg} a \cdot \sec a = \operatorname{cosec} a$

$$\operatorname{cotg} a \cdot \sec a = \frac{\cos a}{\operatorname{sen} a} \cdot \frac{1}{\cos a} = \frac{1}{\operatorname{sen} a} = \operatorname{cosec} a$$

$$\text{e) } \sec^2 a + \operatorname{cosec}^2 a = \frac{1}{\sin^2 a \cdot \cos^2 a}$$

$$\sec^2 a + \operatorname{cosec}^2 a = \frac{1}{\cos^2 a} + \frac{1}{\sin^2 a} = \frac{\sin^2 a + \cos^2 a}{\sin^2 a \cdot \cos^2 a} = \frac{1}{\sin^2 a \cdot \cos^2 a}$$

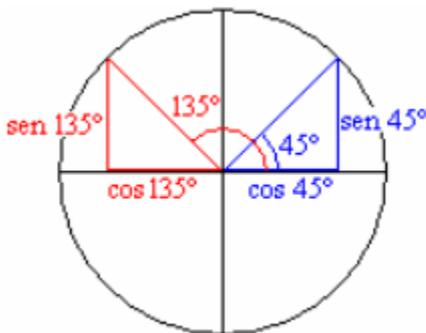
Cuestión 14:

Calcular las razones trigonométricas de los siguientes ángulos en función de sus ángulos asociados agudos.

- a) 135°
- b) 120°
- c) 330°
- d) 240°
- e) 150°
- f) 1290°
- g) Sabiendo que $\operatorname{tg} 18^\circ = 0,32$ calcular las razones trigonométricas de los siguientes ángulos:
 - i) 72°
 - ii) 108°
 - iii) 162°
 - iv) 198°
 - v) 252°
 - vi) 288°
 - vii) 342°

Solución.

a. 135° es suplementario con 45° ($135^\circ + 45^\circ = 180^\circ$). Las razones trigonométricas de 135° están relacionadas con las de 45° , la forma más sencilla de encontrar esta relación es de forma gráfica.

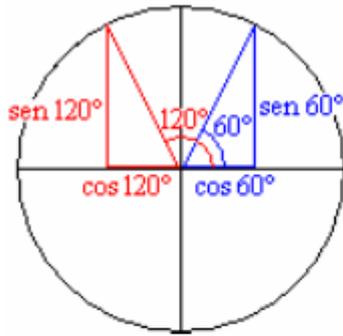


$$\operatorname{sen} 135^\circ = \operatorname{sen} 45^\circ = \frac{\sqrt{2}}{2}$$

$$\operatorname{cos} 135^\circ = -\operatorname{cos} 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\operatorname{tg} 135^\circ = \frac{\operatorname{sen} 135^\circ}{\operatorname{cos} 135^\circ} = \frac{\operatorname{sen} 45^\circ}{-\operatorname{cos} 45^\circ} = -\operatorname{tg} 45^\circ = -1$$

b. 120° es suplementario con 60° ($120^\circ + 60^\circ = 180^\circ$). Las razones trigonométricas de 120° están relacionadas con las de 60° , la forma más sencilla de encontrar esta relación es de forma gráfica.

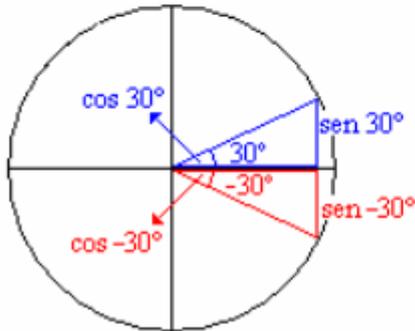


$$\text{sen } 120^\circ = \text{sen } 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{cos } 120^\circ = -\text{cos } 60^\circ = -\frac{1}{2}$$

$$\text{tg } 120^\circ = \frac{\text{sen } 120^\circ}{\text{cos } 120^\circ} = \frac{\text{sen } 60^\circ}{-\text{cos } 60^\circ} = -\text{tg } 60^\circ = -\sqrt{3}$$

c. 330° equivalente a -30° , asociado a 30°

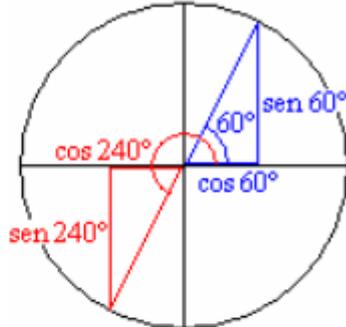


$$\text{sen } -30^\circ = -\text{sen } 30^\circ = -\frac{1}{2}$$

$$\text{cos } -30^\circ = \text{cos } 30^\circ = \frac{\sqrt{3}}{2}$$

$$\text{tg } (-30^\circ) = \frac{\text{sen } (-30^\circ)}{\text{cos } (-30^\circ)} = -\text{tg } 30^\circ = -\frac{\sqrt{3}}{3}$$

d. 240° se asocia a 60° porque se diferencia del él en 180° ($240^\circ = 60^\circ + 180^\circ$).

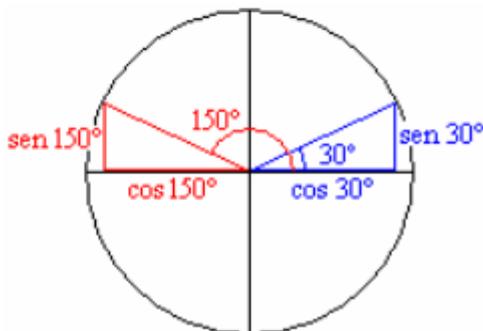


$$\text{sen } 240^\circ = -\text{sen } 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\text{cos } 240^\circ = -\text{cos } 60^\circ = -\frac{1}{2}$$

$$\text{tg } 240^\circ = \frac{\text{sen } 240^\circ}{\text{cos } 240^\circ} = \frac{-\text{sen } 60^\circ}{-\text{cos } 60^\circ} = \text{tg } 60^\circ = \sqrt{3}$$

e. 150° suplementario de 30° ($150^\circ + 30^\circ = 180^\circ$)



$$\text{sen } 150^\circ = \text{sen } 30^\circ = \frac{1}{2}$$

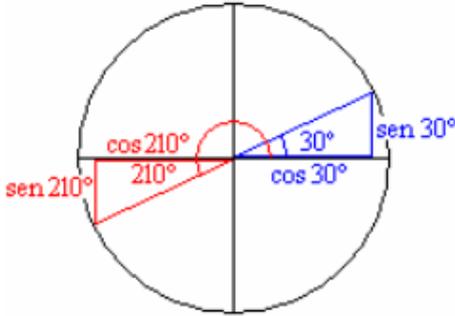
$$\text{cos } 150^\circ = -\text{cos } 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\text{tg } 150^\circ = \frac{\text{sen } 150^\circ}{\text{cos } 150^\circ} = \frac{\text{sen } 30^\circ}{-\text{cos } 30^\circ} = -\text{tg } 30^\circ = -\frac{\sqrt{3}}{3}$$

- f. 1290. Por ser un ángulo superior a 360° , se divide por 360 y nos quedamos con el resto.

$$1290^\circ = 3 \times 360^\circ + 210^\circ$$

Las razones trigonométricas de 1290° coinciden con las de 210° , (relación entre las razones trigonométricas de ángulos que se diferencian en un número entero de vueltas, 360° ó 2π radianes) y las de este (210°) se relacionan con las de 30° ($210^\circ = 180^\circ + 30^\circ$).



$$\text{sen } 1290^\circ = \text{sen}(360^\circ \times 3 + 210^\circ) = \text{sen } 210^\circ = -\text{sen } 30^\circ = -\frac{1}{2}$$

$$\text{cos } 1290^\circ = \text{cos}(360^\circ \times 3 + 210^\circ) = \text{cos } 210^\circ = -\text{cos } 30^\circ = -\frac{\sqrt{3}}{2}$$

$$\text{tg } 1290^\circ = \text{tg}(360^\circ \times 3 + 210^\circ) = \text{tg } 210^\circ = \frac{\text{sen } 210^\circ}{\text{cos } 210^\circ} = \frac{-\text{sen } 30^\circ}{-\text{cos } 30^\circ} = \text{tg } 30^\circ = \frac{\sqrt{3}}{2}$$

- g. Lo primero es calcular el seno y el coseno de 18° conocida la tangente ($\text{tg } 18^\circ = 0,32$). Por ser un ángulo del primer cuadrante, todas sus razones trigonométricas son positivas.

Conocido el valor de la tangente se obtienen la secante.

$$\text{tag}^2 18^\circ + 1 = \text{sec}^2 18^\circ : \text{sec } 18^\circ = \pm \sqrt{\text{tag}^2 18^\circ + 1} = +\sqrt{0,32^2 + 1} = 1,05$$

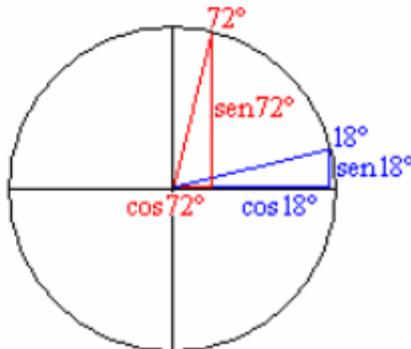
Con la secante se obtiene el coseno

$$\text{sec } 18^\circ = \frac{1}{\text{cos } 18^\circ} : \text{cos } 18^\circ = \frac{1}{\text{sec } 18^\circ} = \frac{1}{1,05} = 0,95$$

Conocidas la tangente y el coseno se obtiene el seno mediante la definición de tangente.

$$\text{tag } 18^\circ = \frac{\text{sen } 18^\circ}{\text{cos } 18^\circ} : \text{sen } 18^\circ = \text{cos } 18^\circ \cdot \text{tag } 18^\circ = 0,95 \cdot 0,32 = 0,30$$

- i. $72^\circ = 90^\circ - 18^\circ$

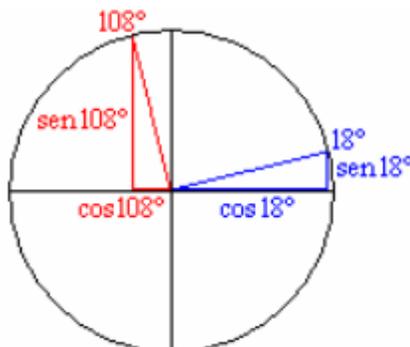


$$\text{sen } 72^\circ = \text{cos } 18^\circ = 0,95$$

$$\text{cos } 72^\circ = \text{sen } 18^\circ = 0,30$$

$$\text{tg } 72^\circ = \frac{\text{sen } 72^\circ}{\text{cos } 72^\circ} = \frac{\text{cos } 18^\circ}{\text{sen } 18^\circ} = \frac{1}{\text{tg } 18^\circ} = \frac{1}{0,32} = 3,12$$

- ii. $108^\circ = 90^\circ + 18^\circ$

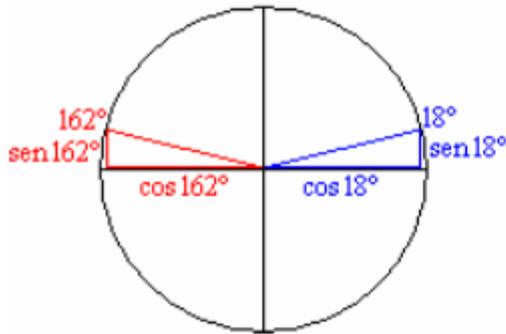


$$\text{sen } 108^\circ = \text{cos } 18^\circ = 0,95$$

$$\text{cos } 108^\circ = -\text{sen } 18^\circ = -0,30$$

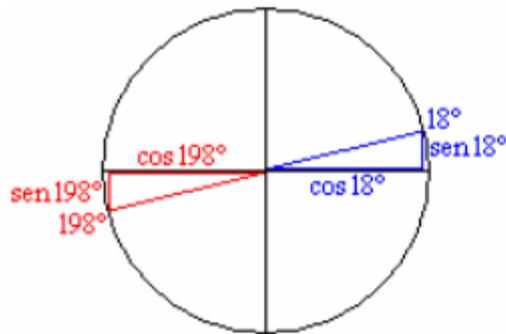
$$\text{tg } 108^\circ = \frac{\text{sen } 108^\circ}{\text{cos } 108^\circ} = \frac{\text{cos } 18^\circ}{-\text{sen } 18^\circ} = -\frac{1}{\text{tg } 18^\circ} = -\frac{1}{0,32} = -3,12$$

iii. $162^\circ = 180^\circ - 18^\circ$



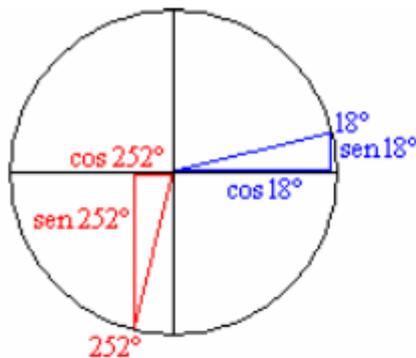
$$\begin{aligned} \text{sen } 162^\circ &= \text{sen } 18^\circ = 0,30 \\ \text{cos } 162^\circ &= -\text{cos } 18^\circ = -0,95 \\ \text{tg } 162^\circ &= \frac{\text{sen } 162^\circ}{\text{cos } 162^\circ} = \frac{\text{sen } 18^\circ}{-\text{cos } 18^\circ} = -\text{tg } 18^\circ = -0,32 \end{aligned}$$

iv. $198^\circ = 180^\circ + 18^\circ$



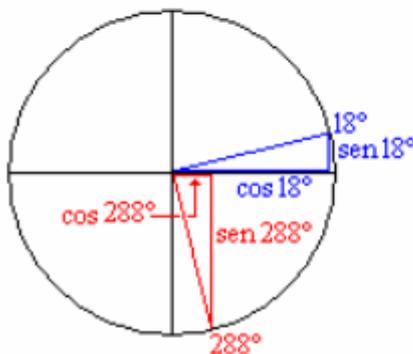
$$\begin{aligned} \text{sen } 198^\circ &= -\text{sen } 18^\circ = -0,30 \\ \text{cos } 198^\circ &= -\text{cos } 18^\circ = -0,95 \\ \text{tg } 198^\circ &= \frac{\text{sen } 198^\circ}{\text{cos } 198^\circ} = \frac{-\text{sen } 18^\circ}{-\text{cos } 18^\circ} = \text{tg } 18^\circ = 0,32 \end{aligned}$$

v. $252^\circ = 270^\circ - 18^\circ$



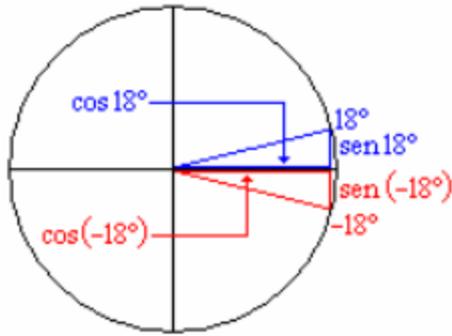
$$\begin{aligned} \text{sen } 252^\circ &= -\text{cos } 18^\circ = -0,95 \\ \text{cos } 252^\circ &= -\text{sen } 18^\circ = -0,30 \\ \text{tg } 252^\circ &= \frac{\text{sen } 252^\circ}{\text{cos } 252^\circ} = \frac{-\text{cos } 18^\circ}{-\text{sen } 18^\circ} = \frac{1}{\text{tg } 18^\circ} = \frac{1}{0,32} = 3,12 \end{aligned}$$

vi. $288^\circ = 270^\circ + 18^\circ$



$$\begin{aligned} \text{sen } 288^\circ &= -\text{cos } 18^\circ = -0,95 \\ \text{cos } 288^\circ &= \text{sen } 18^\circ = 0,30 \\ \text{tg } 288^\circ &= \frac{\text{sen } 288^\circ}{\text{cos } 288^\circ} = \frac{-\text{cos } 18^\circ}{\text{sen } 18^\circ} = -\frac{1}{\text{tg } 18^\circ} = -\frac{1}{0,32} = -3,12 \end{aligned}$$

vii. $342^\circ = -18^\circ$



$$\text{sen}(-18^\circ) = -\text{sen} 18^\circ = -0,30$$

$$\text{cos}(-18^\circ) = \text{cos} 18^\circ = 0,95$$

$$\text{tg}(-18^\circ) = \frac{\text{sen}(-18^\circ)}{\text{cos}(-18^\circ)} = \frac{-\text{sen} 18^\circ}{\text{cos} 18^\circ} = -\text{tg} 18^\circ = -0,32$$

Cuestión 15: Simplificar las siguientes expresiones:

a)

$$\text{sen } \alpha \cdot \frac{1}{\text{tg } \alpha}$$

$$\text{sen } \alpha \cdot \frac{1}{\text{tg } \alpha} = \text{sen } \alpha \cdot \frac{1}{\frac{\text{sen } \alpha}{\text{cos } \alpha}} = \text{sen } \alpha \cdot \frac{\text{cos } \alpha}{\text{sen } \alpha} = \text{cos } \alpha$$

b)

$$\sqrt{1 - \text{sen } \alpha} \cdot \sqrt{1 + \text{sen } \alpha}$$

$$\sqrt{1 - \text{sen } \alpha} \cdot \sqrt{1 + \text{sen } \alpha} = \sqrt{(1 - \text{sen } \alpha) \cdot (1 + \text{sen } \alpha)} = \sqrt{1^2 - \text{sen}^2 \alpha} = \sqrt{\text{cos}^2 \alpha} = \text{cos } \alpha$$

c)

$$\frac{\text{cos}^2 \alpha - \text{sen}^2 \alpha}{\text{cos}^4 \alpha - \text{sen}^4 \alpha}$$

$$\frac{\text{cos}^2 \alpha - \text{sen}^2 \alpha}{\text{cos}^4 \alpha - \text{sen}^4 \alpha} = \frac{\text{cos}^2 \alpha - \text{sen}^2 \alpha}{(\text{cos}^2 \alpha - \text{sen}^2 \alpha) \cdot (\text{cos}^2 \alpha + \text{sen}^2 \alpha)} = \frac{1}{\text{cos}^2 \alpha + \text{sen}^2 \alpha} = \frac{1}{1} = 1$$

d)

$$\frac{\text{cos}^2 \alpha - \text{sen}^2 \alpha}{\text{sen}^2 \alpha - \text{cos}^2 \alpha}$$

$$\frac{\text{cos}^2 \alpha - \text{sen}^2 \alpha}{\text{sen}^2 \alpha - \text{cos}^2 \alpha} = \frac{-(\text{sen}^2 \alpha - \text{cos}^2 \alpha)}{\text{sen}^2 \alpha - \text{cos}^2 \alpha} = -1$$

e)

$$\frac{\text{sec}^2 \alpha + \text{cos}^2 \alpha}{\text{sec}^2 \alpha - \text{cos}^2 \alpha}$$

$$\frac{\sec^2 \alpha + \cos^2 \alpha}{\sec^2 \alpha - \cos^2 \alpha} = \frac{\frac{1}{\cos^2 \alpha} + \cos^2 \alpha}{\frac{1}{\cos^2 \alpha} - \cos^2 \alpha} = \frac{\frac{1 + \cos^4 \alpha}{\cos^2 \alpha}}{\frac{1 - \cos^4 \alpha}{\cos^2 \alpha}} = \frac{1 + \cos^4 \alpha}{1 - \cos^4 \alpha}$$

f)

$$\frac{\operatorname{cosec} \alpha}{1 + \operatorname{ctg}^2 \alpha}$$

$$\frac{\operatorname{cosec} \alpha}{1 + \operatorname{ctg}^2 \alpha} = \frac{\frac{1}{\operatorname{sen} \alpha}}{1 + \frac{\cos^2 \alpha}{\operatorname{sen}^2 \alpha}} = \frac{\frac{1}{\operatorname{sen} \alpha}}{\frac{\operatorname{sen}^2 \alpha + \cos^2 \alpha}{\operatorname{sen}^2 \alpha}} = \frac{\frac{1}{\operatorname{sen} \alpha}}{\frac{1}{\operatorname{sen}^2 \alpha}} = \frac{\operatorname{sen}^2 \alpha}{\operatorname{sen} \alpha} = \operatorname{sen} \alpha$$

g)

$$\frac{\cos^2 \alpha}{1 - \operatorname{sen} \alpha}$$

$$\frac{\cos^2 \alpha}{1 - \operatorname{sen} \alpha} = \frac{1 - \operatorname{sen}^2 \alpha}{1 - \operatorname{sen} \alpha} = \frac{(1 + \operatorname{sen} \alpha) \cdot (1 - \operatorname{sen} \alpha)}{1 - \operatorname{sen} \alpha} = 1 + \operatorname{sen} \alpha$$

Cuestión 16:

Demostrar si son verdaderas o falsas las siguientes ecuaciones:

$$\text{a) } \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta} = \operatorname{tg} \alpha \cdot \operatorname{tg} \beta$$

$$\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta} = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\frac{1}{\operatorname{tg} \alpha} + \frac{1}{\operatorname{tg} \beta}} = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\frac{\operatorname{tg} \beta + \operatorname{tg} \alpha}{\operatorname{tg} \alpha \cdot \operatorname{tg} \beta}} = \operatorname{tg} \alpha \cdot \operatorname{tg} \beta$$

$$\text{b) } \frac{\operatorname{ctg} \alpha + \operatorname{tg} \alpha}{\operatorname{ctg} \alpha - \operatorname{tg} \alpha} = \frac{1}{\cos^2 \alpha - \operatorname{sen}^2 \alpha}$$

$$\frac{\operatorname{ctg} \alpha + \operatorname{tg} \alpha}{\operatorname{ctg} \alpha - \operatorname{tg} \alpha} = \frac{\frac{\cos \alpha}{\operatorname{sen} \alpha} + \frac{\operatorname{sen} \alpha}{\cos \alpha}}{\frac{\cos \alpha}{\operatorname{sen} \alpha} - \frac{\operatorname{sen} \alpha}{\cos \alpha}} = \frac{\frac{\cos \alpha \cdot \cos \alpha + \operatorname{sen} \alpha \cdot \operatorname{sen} \alpha}{\operatorname{sen} \alpha \cdot \cos \alpha}}{\frac{\cos \alpha \cdot \cos \alpha - \operatorname{sen} \alpha \cdot \operatorname{sen} \alpha}{\operatorname{sen} \alpha \cdot \cos \alpha}} = \frac{\cos^2 \alpha + \operatorname{sen}^2 \alpha}{\cos^2 \alpha - \operatorname{sen}^2 \alpha} = \frac{1}{\cos^2 \alpha - \operatorname{sen}^2 \alpha} = \frac{1}{\cos 2\alpha} = \sec 2\alpha$$

$$c) \quad \operatorname{tg} \alpha + \operatorname{ctg} \alpha = \sec \alpha \cdot \operatorname{cosec} \alpha$$

$$\operatorname{tg} \alpha + \operatorname{ctg} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} + \frac{\cos \alpha}{\operatorname{sen} \alpha} = \frac{\operatorname{sen} \alpha \cdot \operatorname{sen} \alpha + \cos \alpha \cdot \cos \alpha}{\cos \alpha \cdot \operatorname{sen} \alpha} = \frac{1}{\cos \alpha \cdot \operatorname{sen} \alpha} = \frac{1}{\cos \alpha} \cdot \frac{1}{\operatorname{sen} \alpha} = \sec \alpha \cdot \operatorname{cosec} \alpha$$

$$d) \quad \operatorname{ctg}^2 \alpha - \cos^2 \alpha = \operatorname{ctg}^2 \alpha \cdot \cos^2 \alpha$$

$$\begin{aligned} \operatorname{ctg}^2 \alpha - \cos^2 \alpha &= \frac{\cos^2 \alpha}{\operatorname{sen}^2 \alpha} - \cos^2 \alpha = \frac{\cos^2 \alpha - \cos^2 \alpha \cdot \operatorname{sen}^2 \alpha}{\operatorname{sen}^2 \alpha} = \frac{\cos^2 \alpha \cdot (1 - \operatorname{sen}^2 \alpha)}{\operatorname{sen}^2 \alpha} = \\ &= \frac{\cos^2 \alpha \cdot \cos^2 \alpha}{\operatorname{sen}^2 \alpha} = \frac{\cos^2 \alpha}{\operatorname{sen}^2 \alpha} \cdot \cos^2 \alpha = \operatorname{ctg}^2 \alpha \cdot \cos^2 \alpha \end{aligned}$$

$$e) \quad \operatorname{sen}^2 \alpha - \cos^2 \beta = \operatorname{sen}^2 \beta - \cos^2 \alpha$$

$$\operatorname{sen}^2 \alpha - \cos^2 \beta = (1 - \cos^2 \alpha) - (1 - \operatorname{sen}^2 \beta) = 1 - \cos^2 \alpha - 1 + \operatorname{sen}^2 \beta = \operatorname{sen}^2 \beta - \cos^2 \alpha$$

$$f) \quad \frac{\operatorname{sen} \alpha \cdot \cos \alpha}{\cos^2 \alpha - \operatorname{sen}^2 \alpha} = \frac{\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\frac{\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} = \frac{\frac{\operatorname{sen} \alpha}{\cos \alpha}}{1 - \frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha}} = \frac{\frac{\operatorname{sen} \alpha}{\cos \alpha}}{\frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{\cos^2 \alpha}} = \frac{\operatorname{sen} \alpha \cdot \cos^2 \alpha}{\cos \alpha \cdot (\cos^2 \alpha - \operatorname{sen}^2 \alpha)} = \frac{\operatorname{sen} \alpha \cdot \cos \alpha}{\cos^2 \alpha - \operatorname{sen}^2 \alpha}$$

$$g) \quad \frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha} = \frac{\cos \alpha + \operatorname{sen} \alpha}{\cos \alpha - \operatorname{sen} \alpha}$$

$$\frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha} = \frac{1 + \frac{\operatorname{sen} \alpha}{\cos \alpha}}{1 - \frac{\operatorname{sen} \alpha}{\cos \alpha}} = \frac{\frac{\cos \alpha + \operatorname{sen} \alpha}{\cos \alpha}}{\frac{\cos \alpha - \operatorname{sen} \alpha}{\cos \alpha}} = \frac{\cos \alpha + \operatorname{sen} \alpha}{\cos \alpha - \operatorname{sen} \alpha}$$

$$h) \quad \cos^2 \alpha \cdot \cos^2 \beta - \operatorname{sen}^2 \alpha \cdot \operatorname{sen}^2 \beta = \cos^2 \alpha - \operatorname{sen}^2 \beta$$

$$\begin{aligned} \cos^2 \alpha \cdot \cos^2 \beta - \operatorname{sen}^2 \alpha \cdot \operatorname{sen}^2 \beta &= \cos^2 \alpha \cdot (1 - \operatorname{sen}^2 \beta) - (1 - \cos^2 \alpha) \cdot \operatorname{sen}^2 \beta = \\ &= \cos^2 \alpha - \cos^2 \alpha \cdot \operatorname{sen}^2 \beta - \operatorname{sen}^2 \beta + \cos^2 \alpha \cdot \operatorname{sen}^2 \beta = \cos^2 \alpha - \operatorname{sen}^2 \beta \end{aligned}$$

$$i) \quad \operatorname{sen} \alpha \cdot \cos \alpha \cdot \operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha \cdot \sec \alpha \cdot \operatorname{cosec} \alpha = 1$$

$$\operatorname{sen} \alpha \cdot \cos \alpha \cdot \operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha \cdot \sec \alpha \cdot \operatorname{cosec} \alpha = \operatorname{sen} \alpha \cdot \cos \alpha \cdot \frac{\operatorname{sen} \alpha}{\cos \alpha} \cdot \frac{\cos \alpha}{\operatorname{sen} \alpha} \cdot \frac{1}{\cos \alpha} \cdot \frac{1}{\operatorname{sen} \alpha} = 1$$

$$\text{j) } (\operatorname{sen} \alpha + \cos \alpha)^2 + (\operatorname{sen} \alpha - \cos \alpha)^2 = 2$$

$$\begin{aligned} (\operatorname{sen} \alpha + \cos \alpha)^2 + (\operatorname{sen} \alpha - \cos \alpha)^2 &= \operatorname{sen}^2 \alpha + 2\operatorname{sen} \alpha \cdot \cos \alpha + \cos^2 \alpha + \operatorname{sen}^2 \alpha - 2\operatorname{sen} \alpha \cdot \cos \alpha + \cos^2 \alpha = \\ &= (\operatorname{sen}^2 \alpha + \cos^2 \alpha) + (\operatorname{sen}^2 \alpha + \cos^2 \alpha) = 1 + 1 = 2 \end{aligned}$$

$$\text{k) } \operatorname{ctg} \alpha - \frac{\operatorname{ctg}^2 \alpha - 1}{\operatorname{ctg} \alpha} = \operatorname{tg} \alpha$$

$$\begin{aligned} \operatorname{ctg} \alpha - \frac{\operatorname{ctg}^2 \alpha - 1}{\operatorname{ctg} \alpha} &= \frac{1}{\operatorname{tg} \alpha} - \frac{\frac{1}{\operatorname{tg}^2 \alpha} - 1}{\frac{1}{\operatorname{tg} \alpha}} = \frac{1}{\operatorname{tg} \alpha} - \frac{1 - \operatorname{tg}^2 \alpha}{\frac{1}{\operatorname{tg} \alpha}} = \frac{1}{\operatorname{tg} \alpha} - \operatorname{tg} \alpha \cdot (1 - \operatorname{tg}^2 \alpha) = \\ &= \frac{1}{\operatorname{tg} \alpha} - \frac{1 - \operatorname{tg}^2 \alpha}{\operatorname{tg} \alpha} = \frac{1 - (1 - \operatorname{tg}^2 \alpha)}{\operatorname{tg} \alpha} = \frac{1 - 1 + \operatorname{tg}^2 \alpha}{\operatorname{tg} \alpha} = \frac{\operatorname{tg}^2 \alpha}{\operatorname{tg} \alpha} = \operatorname{tg} \alpha \end{aligned}$$

$$\text{l) } \frac{1 + \operatorname{tg}^2 \alpha}{\operatorname{ctg} \alpha} = \frac{\operatorname{tg} \alpha}{\cos^2 \alpha}$$

$$\frac{1 + \operatorname{tg}^2 \alpha}{\operatorname{ctg} \alpha} = \frac{1 + \frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha}}{\frac{1}{\operatorname{tg} \alpha}} = \frac{\frac{\cos^2 \alpha + \operatorname{sen}^2 \alpha}{\cos^2 \alpha}}{\frac{1}{\operatorname{tg} \alpha}} = \frac{1}{\frac{\cos^2 \alpha}{\operatorname{tg} \alpha}} = \frac{\operatorname{tg} \alpha}{\cos^2 \alpha}$$

$$\text{m) } \frac{\operatorname{sen} \alpha + \operatorname{ctg} \alpha}{\operatorname{tg} \alpha + \operatorname{cosec} \alpha} = \cos \alpha$$

$$\frac{\operatorname{sen} \alpha + \operatorname{ctg} \alpha}{\operatorname{tg} \alpha + \operatorname{cosec} \alpha} = \frac{\operatorname{sen} \alpha + \frac{\cos \alpha}{\operatorname{sen} \alpha}}{\frac{\operatorname{sen} \alpha}{\cos \alpha} + \frac{1}{\operatorname{sen} \alpha}} = \frac{\frac{\operatorname{sen}^2 \alpha + \cos \alpha}{\operatorname{sen} \alpha}}{\frac{\operatorname{sen}^2 \alpha + \cos \alpha}{\cos \alpha \cdot \operatorname{sen} \alpha}} = \frac{1}{\frac{\operatorname{sen} \alpha}{\cos \alpha \cdot \operatorname{sen} \alpha}} = \frac{\cos \alpha \cdot \operatorname{sen} \alpha}{\operatorname{sen} \alpha} = \cos \alpha$$

$$\text{n) } \frac{1 - \operatorname{sen} \alpha}{\cos \alpha} = \frac{\cos \alpha}{1 + \operatorname{sen} \alpha}$$

$$\frac{1 - \operatorname{sen} \alpha}{\cos \alpha} = \frac{(1 - \operatorname{sen} \alpha) \cdot (1 + \operatorname{sen} \alpha)}{\cos \alpha \cdot (1 + \operatorname{sen} \alpha)} = \frac{1^2 - \operatorname{sen}^2 \alpha}{\cos \alpha \cdot (1 + \operatorname{sen} \alpha)} = \frac{\cos^2 \alpha}{\cos \alpha \cdot (1 + \operatorname{sen} \alpha)} = \frac{\cos \alpha}{1 + \operatorname{sen} \alpha}$$